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EXERCISES IN TIE-BREAKING

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1 FOREWORD

In August 2023, the FIDE Council approved the new regulations for playoff and tiebreak, which contains important changes, as for example the abolition of the virtual opponent and a deep change in the management of unplayed games and matches.

This paper aims to help study the new rules by supplying some practical examples of how they should be applied in a realistic context, for each tie-break system. This is not a paper you want to read or study, but a tool, a collection of exercises to solve after studying, to verify and perfect the comprehension of the rules.

In tournament practice, tie-breaking is carried out with a sequence of methods applied in succession until all the ties are resolved; here, for simplicity, we will always use the chosen system as the first tie-breaker.

As a methodology, the reader might want to follow the exercises in progression, in the order in which they are presented, keeping at hand a copy of the crosstable and of the FIDE regulations on technical play-offs and tie-breaks (C.07), to which all references in brackets (“[]”) refer. It would be advisable to try to solve the exercise by yourself, and then compare the procedure and calculations with the given solution and the rules.

This paper aims to be accessible even to beginners, as long as they know the rules well enough that they can find there the basic ideas that the exercises illustrate, which we tried to introduce gradually. Even the more expert readers will probably find it advisable to read, at least briefly, even the simplest examples - even if they are already well familiar with the subject - so as not to lose any useful information.

We conclude by observing that C.07 regulations try to consider all the tie-breaks used in chess, even if the value of some of them may be questionable. The event organizer should choose which ones to use, or even possibly invent new ones.

Enjoy and good luck!

2 THE CROSSTABLES

In order to help comparing the various systems, we use the same tournaments in all the examples, namely a Swiss and a Round-robin for individuals, and a Swiss for teams. The scoring system is the traditional one (zero points for a loss, half a point for a draw, one point for a win for single games; zero points for a loss, one for a draw and two for a win for team matches).

2.1 Swiss individual tournament crosstable

For convenience, the board is sorted by score (descending) and pairing number (i.e., initial ranking). Unplayed games are highlighted in red: the half-point requested byes (HPB) of David (#4, second round) and Jessica (#9, third round); the forfeit defeats of Paul (#14, third round) and Jessica (#9, fourth round); the justified absence (zero points bye, ZPB) of Paul (#14, fourth round); the withdrawal of Nick (#12, fourth round); the byes received from the pairing (PAB). Opponents with any kind of unplayed games are highlighted too (yellow background).

#	NAME	ELO	SCORE	1	2	3	4	5
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5

2.2 Round-robin individual tournament crosstable

For our convenience, this crosstable is presented both in the format traditionally used for round-robin tournaments and in the format typically used for Swiss tournaments. The first format highlights the forfeit defeat of Franck (#6) in the fourth round, which in the "Swiss" format is not distinguishable because forfeit wins and losses are equivalent to played games in round-robin tournaments.

#	NAME	ELO	SCORE	1	2	3	4	5	6
1	Alyx	2200	3.5	*	1	1	0	1	½
2	Bruno	2150	3.5	0	*	½	1	1	1
3	Charline	2100	3.5	0	½	*	1	1	1
4	David	2050	1.5	1	0	0	*	½	0
5	Helene	2000	1.5	0	0	0	½	*	+
6	Franck	1950	1.5	½	0	0	1	-	*

#	NAME	ELO	SCORE	1	2	3	4	5
1	Alyx	2200	3.5	=W5	+W2	+B3	-W4	+B6
2	Bruno	2150	3.5	+W6	-B1	+W5	=W3	+B4
3	Charline	2100	3.5	+W4	+B6	-W1	=B2	+W5
4	David	2050	1.5	-B3	-B5	=W6	+B1	-W2
5	Franck	1950	1.5	=B1	+W4	-B2	-W6	-B3
6	Helene	2000	1.5	-B2	-W3	=B4	+B5	-W1

2.3 Swiss team tournament crosstable

Team tournaments have some characteristics that distinguish them from individual ones. The first and foremost is that each team has two scores, one relating to the points obtained in the match ("Match points", MP), and one relating to the players' points ("Game points", GP) [11.1]. Our Swiss tournament includes 14 teams, fielding 4 players each (credits: Roberto Ricca). We need the crosstables (individual and team) and the composition of each team; all this data is usually made available by the pairing software.

#	TEAM	MP	GP	R1	R2	R3	R4	R5	R6	R7
1	Antelopes	10	17,5	8w2,5	4b1,5	7w3	2b2,5	5w1,5	+F	3b2,5
2	Bonobos	10	17	9b3	3w4	5b0	1w1,5	13w3	4b2,5	10b3
3	Cougars	10	16	10w2,5	2b0	9w3	6b2,5	4w3	5b3,5	1w1,5
4	Deer	10	17	11b3	1w2,5	13b3	5w3,5	3b1	2w1,5	9b2,5
5	Elephants	10	18	12w4	6b3	2w4	4b0,5	1b2,5	3w0,5	13b3,5
6	Falcons	7	12,5	13b2	5w1	8b3	3w1,5	11b2,5	-F	12w2,5
7	Giraffes	6	11,5	14w2	8b2	1b1	10w2,5	9b2	13w2	ZPB
8	Hippopotami	7	15	1b1,5	7w2	6w1	12b3,5	10b2	9w2	11w3
9	Iguanas	6	14,5	2w1	12b4	3b1	14w3	7w2	8b2	4w1,5
10	Jackals	5	13	3b1,5	11w1,5	12b2,5	7b1,5	8w2	14b3	2w1
11	Koalas	4	11,5	4w1	10b2,5	14w2	13b1,5	6w1,5	12w2	8b1
12	Lynxes	2	7,5	5b0	9w0	10w1,5	8w0,5	PAB	11b2	6b1,5
13	Moose	6	11,5	6w2	14b2,5	4w1	11w2,5	2b1	7b2	5w0,5
14	Narwhals	4	11,5	7b2	13w1,5	11b2	9b1	HPB	10w1	PAB

#	TEAM	PLAYERS					
		1	2	3	4	5	6
1	Antelopes	1	14	16	39	50	64
2	Bonobos	2	29	21	26	65	46
3	Cougars	3	25	32	20	58	54
4	Deer	4	11	9	68	48	70
5	Elephants	10	22	27	35	59	72
6	Falcons	18	13	41	38	53	63
7	Giraffes	5	17	44	45	42	77
8	Hippopotami	28	19	24	49	66	60
9	Iguanas	8	12	31	79	61	67
10	Jackals	6	30	47	57	74	83
11	Koalas	33	23	43	52	78	73
12	Lynxes	15	40	51	69	75	84
13	Moose	7	56	36	80	76	82
14	Owls	34	55	37	62	71	81

ID	T	PLAYER	ELO	NP	PT	R1	R2	R3	R4	R5	R6	R7
1	1	Armando Aglio	2110	5	6,0	28w1		05w1	02b1	10w1	18b+	03b1
2	2	Bernardo Berci	2105	6	3,0	08b1	03w1	10b0	01w0		11b0	30b1
3	3	Carlo Cohen	2100	7	4,0	06w1	02b0	08w0	18b1	04w1	10b1	01w0
4	4	Diego De La Vega	2096	4	2,0	33b0		56b1		03b0		08b1
5	7	Gregor Gewiss	2091	5	2,0	34w=	28b=	01b0	06w1	08b0		ZPB
6	10	John Jelba	2086	6	3,5	03b0	33w=	15b1	05b0	28w1	34b1	
7	13	Marius Montana	2081	5	2,5	13w1			33w1	29b=	17b0	10w0
8	9	Isidora Iago	2076	7	5,0	02w0	15b1	03b1	34w1	05w1	28b1	04w0
9	4	Desiree Dong	2071	4	2,5		16b1		22b1	32b=	29b0	
10	5	Elizabeth Era	2066	7	4,0	15w1	18b1	02w1	11b0	01b0	03w0	07b1
11	4	David Dong	2061	6	2,5		14w0	80w0	10w1	25w=	02w1	12w0
12	9	Ismaele Imbotto	2056	5	3,5		40w1		55b1	44b=	19w0	11b1
13	6	Frank Fala	2051	5	3,5	07b0		24w1	25b1	23w1	14b-	40b=
14	1	Alejandro Almeida	2046	6	5,0	19b=	11b1	17b1	29w0	22b=	13w+	25w1
15	12	Lucas Locas	2041	4	0,5	10b0	08w0	06w0		PAB		18b=
16	1	Alcide Angolano	2036	3	1,5		09w0		65b1			32b=
17	7	Genny Gewiss	2031	5	2,5	55b1	19w=	14w0	30b0		07w1	ZPB
18	6	Filippa Franceschi	2026	5	1,5		10w0	28b0	03w0	33b1	01w-	15w=
19	8	Herbert Honacek	2021	6	4,0	14w=	17b=		51w=	30w=	12b1	43b1
20	3	Cosimo Chespari	2016	3	2,0		26b0	61w1	53w1			
21	2	Barbara Bernard	2011	3	1,5			35b0		36b=	68b1	
22	5	Elsa Era	2006	7	4,5	40b1	41w1	29b1	09w0	14w=	25b=	56w=
23	11	Kristin Kormans	2001	5	0,0	68b0	30w0		56w0	13b0	51b0	
24	8	Helmut Holler	1997	6	2,5	39b0	44w=	13b0	69b1		31w=	52w=
25	3	Cristian Celamont	1992	6	1,0	30b0	29w0		13w0	11b=	22w=	14b0
26	2	Bruno Boita	1987	4	3,5	79b1	20w1			80w1		57b=
27	5	Erika Espate	1982	1	1,0							80b1
28	8	Hans Holz	1977	7	3,0	01b0	05w=	18w1	40b1	06b0	08w0	33w=
29	2	Bruce Belanoy	1972	7	5,5	31w1	25b1	22w0	14b1	07w=	09w1	47w1
30	10	Jean Joyce	1967	6	4,5	25w1	23b1	40w1	17w1	19b=		02w0
31	9	Ingrid Ilvas	1962	5	2,0	29b0		32w0	37w1	42w=	24b=	
32	3	Cesira Cohen	1957	6	4,5	57w1		31b1	38b=	09w=	35b1	16w=
33	11	Kris Kelpa	1952	7	3,5	04w1	06b=	34w1	07b0	18w0	40w=	28b=
34	14	Nikola Neric	1947	4	0,5	05b=		33b0	08b0	HPB	06w0	PAB
35	5	Enza Eliprandi	1942	5	3,5			21w1	68b=	39b1	32w0	76w1
36	13	Marino Marino	1937	4	2,0	38w=	37w=		43w=	21w=		

37	14	Nicola Neba	1932	3	1,0		36b=	52b=	31b0	HPB		PAB
38	6	Francisco Formenteros	1927	5	3,0	36b=	59w=	66w1	32w=	73w=	50b-	
39	1	Abel Adardo	1922	5	2,5	24w1	68b0	45w0	46w=	35w0	41b+	
40	12	Lonnie Lemmie	1917	6	1,0	22w0	12b0	30b0	28w0	PAB	33b=	13w=
41	6	Federico Frappani	1912	5	3,0	56w1	22b0	49b1		52b0	39w-	51w1
42	7	George Gotham	1907	4	1,0	81b0		50w0	74b=	31b=		ZPB
43	11	Kelly Kort	1902	5	2,0	48w0		55b=	36b=		69w1	19w0
44	7	Gipo Gressi	1897	4	2,5	71w=	24b=			12w=	56b1	ZPB
45	7	Gunnar Gunnarson	1892	4	2,5		66w=	39b1	47w1		80w0	ZPB
46	2	Blanca Bolaverde	1887	4	1,5	61w0		59w0	39b=	82b1		
47	10	June Joyce	1882	5	1,0			51b0	45b0	49w0	55w1	29b0
48	4	Delly Drago	1877	6	4,5	43b1	50b=	82w1	59b1		65b=	61w=
49	8	Hakim Ho	1872	4	3,0	50w1		41w0		47b1		78b1
50	1	Asdrubale Arca	1867	3	2,5	49b0	48w=	42b1			38w+	
51	12	Leone Leone	1862	4	2,5			47w1	19b=	PAB	23w1	41b0
52	11	Kirk Koman	1857	6	4,5		57b1	37w=	76w1	41w1	75b=	24b=
53	6	Flavio Federici	1852	3	1,0		72b=		20b0			75b=
54	3	Cammy Calat	1847	2	2,0						59w1	64b1
55	14	Noah Negus	1842	5	0,5	17w0	56w0	43w=	12w0	HPB	47b0	PAB
56	13	Maurice Melancon	1837	6	2,5	41b0	55b1	04w0	23b1		44w0	22b=
57	10	Juan Jupp	1832	4	0,5	32b0	52w0				62b0	26w=
58	3	Cory Cniser	1827	4	2,5	74b=	65w0	67b1		68b1		
59	5	Emanuelle Ener	1822	5	2,5	69w1	38b=	46b1	48w0		54b0	
60	8	Hannibal Hermol	1817	2	1,5				75w1	74w=		
61	9	Ivo Ierasimov	1812	4	2,5	46b1	75w1	20b0				48b=
62	14	Nuccio Negri	1807	4	3,0		80w0	73w1	79w1	HPB	57w1	PAB
63	6	Frederick Fogar	1802	1	0,5	82w=						
64	1	Aaron Asta	1797	2	0,0					72b0		54w0
65	2	Brandon Bogart	1792	4	2,0		58b1		16w0		48w=	74w=
66	8	Hyeronimus Hermol	1787	3	1,0		45b=	38b0			79b=	
67	9	Ion Iodiad	1782	2	0,0			58w0		77b0		
68	4	Daria De La Vega	1777	7	4,5	23w1	39w1	76b1	35w=	58w0	21w0	79b1
69	12	Lydia Lameran	1772	4	0,0	59b0	79w0		24w0	PAB	43b0	
70	4	Diana Drago	1767	1	1,0	78w1						
71	14	Norberto Nodo	1762	2	1,5	44b=	82b1			HPB		PAB
72	5	Erika Ecore	1757	3	2,5	84b1	53w=			64w1		
73	11	Kurt Kontos	1752	3	1,5		74w1	62b0		38b=		
74	10	Julio Joyce	1747	7	3,5	58w=	73b0	75w=	42w=	60b=	81w1	65b=
75	12	Lavinia Lentrero	1742	5	1,5		61b0	74b=	60b0	PAB	52w=	53w=
76	13	Michael Morte	1737	3	0,0			68w0	52b0			35b0
77	7	Gennady Gomirov	1732	2	1,0					67w1	82b0	ZPB
78	11	Ky Korbel	1727	2	0,0	70b0						49w0
79	9	Isobel Iodiad	1722	5	1,5	26w0	69b1		62b0		66w=	68w0
80	13	Manuel Malagracia	1717	5	3,0		62b1	11b1		26b0	45b1	27w0
81	14	Nando Nodo	1712	2	1,0	42w1				HPB	74b0	PAB
82	13	Marko Mokala	1707	5	1,5	63b=	71w0	48b0		46w0	77w1	
83	10	Jacques Junipero	1702	0	0,0							
84	12	Lana Leva	1697	1	0,0	72w0				PAB		

Unplayed matches are highlighted for clarity. There are no universal rules for the management of such matches. The treatment rules should therefore be provided for by the specific tournament regulations. Here are the provisions for our tournament:

- PAB: one match point, two game points and zero player points
- HPB: one match point, two game points and zero player points
- ZPB: zero match points, zero game points and zero player points

- -F: zero match points, zero game points and zero player points
- +F: two match points, 4 game points, one point for each player

Note: The sum of the individual scores of the players may differ from the game points score (GP) of the team, because some types of unplayed matches (PAB, HPB, ...) give game points to the team but do not give points to the players.

For some tie-breaks, we also need the exact order of players fielding. Such information should be deduced from pairings, a task that is not difficult but rather tedious; this data will be provided only as needed.

PART ONE –INDIVIDUAL TOURNAMENTS

3 TIE-BREAKS BASED ON BUCHHOLZ SYSTEM AND SIMILAR

The biggest difficulty encountered in calculating Buchholz scores comes from unplayed games, either of the player or of some opponent of theirs. In the following exercises, those concepts are introduced gradually to aid their assimilation.

This chapter does not include round-robin exercises because, as is known, Buchholz-type tie-breaks are not applicable to this type of tournaments.

Buchholz in round-robin tournaments

Buchholz is useless in a round-robin tournament, because it cannot break ties (all players with the same score have the same tiebreaker). In fact, in these tournaments each player meets all other players (any games won or lost by forfeit are equivalent to played games). If the tournament includes N players (for simplicity, let's assume N even; the extension to the odd case is in any case immediate), a total of $\frac{1}{2}N \cdot (N-1)$ games take place, each of which distributes one point; so (except in exceptional cases) this is also the total of points distributed among all players (for example, a tournament with 6 players distributes $6 \times 5 / 2 = 15$ points). The player's Buchholz is the sum of the opponents' scores - therefore, to calculate it, we simply subtract the player's score from the total (e.g., if they scored 3 points, their Buchholz is $15 - 3 = 12$).

3.1 Buchholz (total)

Exercise 1

In the Swiss tournament, calculate the Buchholz score (total) of player #2.

Let's consider the following excerpt from the crosstable, showing the tournament history of the player (first row, in blue) and of all their opponents, and their scores:

#	NAME	ELO	SCORE	1	2	3	4	5
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5

Since neither the player nor any of the opponents have any unplayed games, the basic Buchholz definition [8.1] applies, and no special adjustments are required; the player's Buchholz is simply given by the sum of the opponents' scores:

$$BH(\#2) = 1.0 + 1.5 + 3.5 + 3.5 + 3.5 = 13.0.$$

Exercise 2

In the Swiss tournament, determine the ranking order between players #1 and #3 using the Buchholz system.

From the crosstable, let's extract the tournament histories for the involved players.

#1	NAME	ELO	SCORE	1	2	3	4	5
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

#3	NAME	ELO	SCORE	1	2	3	4	5
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6

Both played all their games, but the same is not true for all their opponents. We should therefore calculate the opponents' contribution taking into account their unplayed games.

Let's start with #1 (Alyx) opponents:

Opponent	+W9 (*)	=B13	=W2	+B15	=W4 (*)
Opp. score	1.5	1.5	4.0	2.0	3.5

Opponents highlighted with an asterisk have one or more unplayed games, and their score may need to be corrected. To do this, let's take a look at their tournament history, which includes several types of unplayed games. The score of a player who has unplayed matches must be adjusted differently (as already happened with previous regulations) depending on whether it is used to calculate the tie-break of the players themselves or their opponents. At the moment we are interested only in the second case – in fact, to calculate the Buchholz of player #1, we need to adjust the scores of players #4 and #9.

#	NAME	ELO	SCORE	1	2	3	4	5
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE

For this calculation, we must resort to articles 16.2-16.3 of the regulations. The adjustment takes place only for the rounds in which the player *requested a bye* (those in which the player was possibly withdrawn also count as such) *and which were not followed by any round in which the player was available to play* [16.1.2]. For the purposes of the opponents' Buchholz, these rounds count as draws [16.3.2]. In all other cases, the

unplayed round is calculated with the assigned score [16.3.1], i.e., the same one displayed on the crosstable and added to obtain the total score, which, for convenience, from now on we will call *face value*.

For both players #4 and #9, all unplayed rounds are followed by at least one round in which the player was available to play, and are therefore calculated at their face value. This basically means that we can calculate the Buchholz of player #1 simply by adding all the scores of their opponents as they are shown on the scoreboard. We therefore have: $BH(\#1) = 1.5+1.5+4.0+2.0+3.5=12.5$.

Let's now calculate the tie-break of player #3 (Charline). Again, we have to investigate the opponents who had unplayed games, which are #4, #6 and #11.

#3	NAME	ELO	SCORE	1	2	3	4	5
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6

Once again, all unplayed rounds are followed by at least one round with availability to play, so all results are taken at face value. We can therefore calculate the Buchholz, which is $BH(\#3) = 2.5+3.0+2.5+3.5+4.0=15.5$.

Hence, player #3 (Charline) precedes #1 (Alyx) in the rankings.

Exercise 3

In the Swiss tournament, determine the ranking order between players #5, #8 and #11 using the Buchholz system.

We now have a new case: one of the players (#11) has an unplayed game, of which we should determine the contribution, not only for opponents as in the previous exercise, but also for the player himself [16.4]. Let's look at the players' data.

#5	NAME	ELO	SCORE	1	2	3	4	5
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5

Player #5 has no unplayed games. Among his opponents, player #11 has a forfeit win (due opponent's forfeit) in the fourth round, which is an unplayed game of the type [16.2.2] and is therefore counted at face value [16.3.1]. Their Buchholz is therefore

$BH(\#5) = 2.5+2.0+1.5+1.5+1.0=8.5$.

#8	NAME	ELO	SCORE	1	2	3	4	5
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Also Player #8 has no unplayed games, and among the opponents we count a bye assigned by the pairing (PAB) [16.2.1], a forfeit defeat [16.2.4], and a round of scheduled absence (i.e., a zero points bye on request) followed by a played round [16.2.3]. All these results are calculated at face value, so Buchholz is $BH(\#8) = 3.5+3.5+3.0+2.0+1.5=13.5$.

#11	NAME	ELO	SCORE	1	2	3	4	5
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11

Finally, player #11's opponents only have regular played games, but this time it is the player themselves who did not play a round, namely because of a forfeit win [16.2.2]. For this round, the contribution that the player gives to themselves is calculated as a game played against a dummy opponent (*not to be confused with the virtual opponent!*) who has the same number of points as the player, and ends with the result corresponding to the points awarded [16.4].

Note: the score of the dummy opponent is not the one held by the player at the time of the unplayed round, but rather the one with which the player finished the tournament. For Buchholz, knowing how the match ended is not relevant, because Buchholz simply adds up the scores of all the player's opponents - but it would be very relevant in the case of the Sonneborn-Berger system!

Now, the dummy opponent has 2.5 points, the very same as player #11. So the player's Buchholz is $BH(\#11) = 3.5+3.5+2.5+2.5+1.5 = 13.5$.

Finally, let's compare the results: player #5, with Buchholz equal to 8.5, is ranked third, while players #8 and #11, both with Buchholz 13.5, are still tied for the first place. To break this tie, we will have to move on to the next tie-break of the list, if any, or to a drawing of lots [4.2].

After these first exercises, we can observe that, in practice, despite the presence of unplayed rounds, the Buchholz of the players involved was calculated simply using the scores of the opponents just as they were displayed in the rankings. This is certainly not a coincidence: *the philosophy of the new regulations is that, in general, the score used for the tie-breaks should be the same one that determines the ranking.* An exception is made

for opponents who finished their tournament prematurely – because they could otherwise damage their opponents' tie-break with an unduly low score (think for example of a player who, after winning several games, was suddenly forced to withdraw for some reason or other). Therefore, to limit the damage to opponents, all unplayed games from the withdrawal onwards are calculated as draws; we will see that in the next exercise.

Exercise 4

In the Swiss tournament, using the Buchholz system, determine the ranking order for players at 3.5 points.

Among the opponents of the affected players, we now find player #12 who, having received a PAB in the second round and, immediately after that, a forfeit win in the third round, withdrew from the tournament from the fourth round. Let us proceed step by step.

#1	NAME	ELO	SCORE	1	2	3	4	5
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Let's start with player #1, who played all his games; among their opponents, we find #4, who got a half-point bye, and #9, who has three unplayed games. The first of these unplayed games, a half-point requested bye, is followed by a forfeit loss (both those are rounds *without availability to play*) [16.2.4], but in the end the player re-entered the tournament, although receiving a PAB [16.2 .1], and was therefore available to play. Hence, the bye on request constitutes an unplayed round of type [16.2.3].

The same goes for player #4, whose bye on request is also followed by played rounds. Ultimately, all unplayed games are calculated at their face value [16.3.1]. The player's Buchholz is therefore $BH(\#1) = 4.0+3.5+2.0+1.5+1.5=12.5$.

#3	NAME	ELO	SCORE	1	2	3	4	5
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7

Let's now look at the situation for player #3. Among their opponents, we count a half-point bye on request [16.2.3], a PAB [16.2.1] and a forfeit win [16.2.2]; once again, all these unplayed games are counted at nominal value, so the Buchholz is

$$BH(\#3) = 4.0+3.5+3.0+2.5+2.5 = 15.5.$$

#16	NAME	ELO	SCORE	1	2	3	4	5
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11

Now let's move on to #16. The situation is similar: among the opponents there is only one forfeit win [16.2.2], and Buchholz is $BH(\#16) = 4.0+2.5+2.5+2.0+1.5=12.5$.

#4	NAME	ELO	SCORE	1	2	3	4	5
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Last, player #4 got a half-point bye, and faced a player who later withdrew. Let's see the contributions due to these rounds.

The half-point bye on request provides the player with a *contribution equal to their score*, i.e., 3.5 points, as per the crosstable.

The contribution of the withdrawn opponent is calculated as follows: the PAB and the forfeit win contribute their nominal value [16.3.1], therefore one point each. The last two rounds, however, which are voluntarily unplayed rounds, and are not followed by any rounds with availability to play, fall into category [16.2.5] and therefore carry a contribution for the opponent of half a point each [16.3.2]. Hence, player #12's adjusted contribution is worth a total of $0.0+1.0+1.0+0.5+0.5 = 3.0$ points.

The Buchholz of #4 is therefore $BH(\#4) = 3.0+3.5+1.5+3.5+3.5 = 15.0$.

The ranking therefore is #3(15.5), #4(15.0) and, still tied, #1 and #16 (12.5). For the last two players we will have to continue with the next tie-breaks or drawing of lots.

3.2 Buchholz Cut-1

Modifiers [14] are rules that alter the behaviour of a tie-break system in a predefined manner; the best known, and most used, is the Cut-1 modifier [14.1], which ignores the least significant contribution in the tie-break calculation.

In the case of the Buchholz system, this usually means *ignoring the contribution of the opponent with the smallest score*; However, if the player has some *voluntarily* unplayed games (forfeit or bye on request), the smallest of the contributions due to these unplayed games will be ignored [16], with the idea that they are less significant than those results that were decided on-the-board.

Exercise 5

In the Swiss tournament, using the Buchholz Cut-1 system, determine the ranking order for players at 2.5 points.

Here we encounter for the first time the application of the Cut-1 modifier [14.1] to the Buchholz system. It requires ignoring, among all the opponents' contributions, the least significant one, which corresponds to the opponent with the lowest score [14.1.1], except when the player has voluntarily unplayed games (this case will be explained later). We proceed as usual to calculate all the opponents' contributions for each player; now, however, before performing the actual sum, we must *identify and discard* the least significant contribution.

#5	NAME	ELO	SCORE	1	2	3	4	5
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5

Player #5 played all their rounds. Among their opponents, #11 has a forfeit win [16.2.2], which is calculated at face value [16.3.1]. The contributions are therefore 2.5, 2.0, 1.5, 1.5, 1.0; the least significant is that of opponent #10, who has fewer points (1.0); the player's Buchholz Cut-1 is therefore $BH-C1(\#5) = 2.5+2.0+1.5+1.5 = 7.5$.

#8	NAME	ELO	SCORE	1	2	3	4	5
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Same holds for player #8, so the received contributions are 3.5, 3.5, 3.0, 2.0, 1.5; among these, the least significant is 1.5, therefore $BH-C1(\#8) = 3.5+3.5+3.0+2.0 = 12.0$.

#11	NAME	ELO	SCORE	1	2	3	4	5
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11

Player #11, on the other hand, has an unplayed game. This is however a forfeit win [16.2.2], and therefore an available-to-play round; the contribution of this round is equal to the player's final score, i.e., 2.5 points. The contributions are therefore 3.5, 3.5, 2.5, 2.5, 1.5. Among these, since there are no voluntarily unplayed rounds (forfeit defeats or byes on request) the smallest one is ignored, namely that of #7 (1.5). Therefore,

$$BH-C1(\#11) = 3.5+3.5+2.5+2.5 = 12.0.$$

The ranking order is therefore #8 and #11 still tied (12.0), #5 (7.5).

Exercise 6

In the Swiss tournament, using the Buchholz Cut-1 system, determine the ranking order for players at 1.5 points.

In this exercise, for the first time we apply the Cut-1 modifier [14.1] for a player who has some unplayed games in his tournament history. For this player we are to apply [16.5] and choose the contribution to ignore among those relating to unplayed games.

As usual, let's look at the players one by one.

#7	NAME	ELO	SCORE	1	2	3	4	5
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16

Player #7 played all his games; subtracting the 2.0 contribution of #15, which is the least significant, we obtain for the Buchholz the value $BH-C1(\#7) = 4.0+3.5+2.5+2.5 = 12.5$.

#9	NAME	ELO	SCORE	1	2	3	4	5
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5

Here too we have nothing new regarding the contributions of the opponents, but player #9 also has three unplayed games that we should examine. We must choose the contribution to be ignored among the games lost by forfeit and byes on request [16.5] – in our case, between the third and fourth rounds (the fifth round is in fact a PAB, and therefore with availability to play, and as such is not among those rounds to be cut first).

For each unplayed game, the contribution is equal to a dummy opponent with the player's score, i.e., 1.5 points. We must discard the lesser contribution due to a voluntarily unplayed round (rounds 3 and 4); being equal, we discard any of the two. The tie-break value is therefore $BH-C1(\#9) = 3.5+1.0+1.5+1.5 = 7.5$.

#13	NAME	ELO	SCORE	1	2	3	4	5
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13

Finally, for #13 again there is nothing special; the least significant contribution is the 2.0 given by opponent #14, so $BH-C1(\#13) = 3.5+3.5+2.5+2.5 = 12.0$.

The ranking order is therefore #7 (12.5), #13 (12.0), #9 (7.5).

Exercise 7

In the Swiss tournament, using the Buchholz Cut-1 system, determine the ranking order for players at 3.5 points.

As usual, let's examine affected players one by one.

#1	NAME	ELO	SCORE	1	2	3	4	5
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Player #1 played all their games, and their opponents' unplayed games must all be calculated at face value [16.3.1]; discarding the least significant contribution (1.5), the tie-break is $BH-C1(\#1) = 4.0+3.5+2.0+1.5 = 11.0$.

#3	NAME	ELO	SCORE	1	2	3	4	5
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7

Same holds for player #3. Discarding the less significant contribution (2.5), we get $BH-C1(\#3) = 4.0+3.5+3.0+2.5 = 13.0$.

#4	NAME	ELO	SCORE	1	2	3	4	5
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Opponent #12 has two unplayed games in the second and third round, respectively of type [16.2.1] and [16.2.2], that are taken at face value [16.3.1]. In the last two rounds, that player withdrew (voluntarily unplayed rounds). For the opponents' Buchholz calculation purposes, each of these rounds is worth as much as a draw [16.3.2], so the total contribution of #12 to the opponents' Buchholz is $0.0+1.0+1.0+0.5+0.5 = 3.0$.

Player #4 requested a half-point bye, which is however followed by rounds with availability to play and must therefore be calculated as a game (a drawn one, because it is an HPB) against a dummy opponent at 3.5 points, i.e., as many as the player themselves [16.4]. However, this is a bye on request, and must therefore be the first contribution to be discarded by the Cut modifiers [16.5].

The Buchholz value is therefore $BH-C1(\#4) = 3.5+3.5+3.0+1.5 = 11.5$.

Note: discarding an unplayed round seems to be a disadvantage for the player (by cutting the minimum contribution, here we would get a $BH-C1=12.5$); in fact, *this rule avoids an unfair advantage* as, without it, a player could skip a round they know they will lose for the sole purpose of improving their Buchholz.

#16	NAME	ELO	SCORE	1	2	3	4	5
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11

The last player played all their games. Among the opponents' unplayed rounds there is only one forfeit win, which is calculated at face value. Discarding the least significant contribution (1.5) we obtain $BH C1(\#16) = 4.0+2.5+2.5+2.0 = 11.0$.

Summarising, the ranking order is #3 (13.0); #4 (11.5); #1, #16 still tied (11.0).

Exercise 8

In the Swiss tournament, using the Buchholz Cut-1 system, determine the ranking order for players at 2.0 points.

In this exercise we find a situation typical of the low ranking, with several types of unplayed games of different kind. As usual, let's examine the players one by one.

#12	NAME	ELO	SCORE	1	2	3	4	5
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1

The contribution from opponent #4's is calculated at face value, as the requested bye in the second round is followed by rounds with availability to play [16.2.3]. Player #12, however, has four unplayed rounds, each of which is calculated as a match with a dummy opponent at 2.0 points, equal to the player's own score [16.4]. One of the contributions from rounds 4 and 5 will be cut (it makes no difference which one, since they are equal), so $BH-C1(\#12) = 3.5+2.0+2.0+2.0 = 9.5$.

#14	NAME	ELO	SCORE	1	2	3	4	5
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Once again, the player has unplayed rounds, both valued at 2.0 points, and one of these will be discarded by Cut-1; therefore $BH-C1(\#14) = 3.0+2.5+2.0+1.5 = 9.0$.

#15	NAME	ELO	SCORE	1	2	3	4	5
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5

The last player has no unplayed game; excluding the least significant contribution (1.0), we obtain $BH-C1 = 3.5+3.5+2.5+1.5 = 11.0$.

The ranking order is therefore #15 (11.0), #12(9.5), #14(9.0).

3.3 Average Buchholz of opponents

The calculation of this tie-break is somewhat more laborious than the previous ones, as it requires the calculation of the Buchholz of all the opponents of the tied players. We can calculate it in two phases: first, we calculate all the necessary Buchholz values and put them in a table; then we calculate averages. For this tie-break, no modifiers are allowed (see rule [5]).

Exercise 9

In the Swiss tournament, using the Average Buchholz of Opponents (AOB) system, determine the tiebreaker and ranking order for all players.

In this exercise we need to calculate the Buchholz of all players (the reader may want to consider this a useful refresher). First, we should calculate the score adjusted for unplayed rounds. In practice, the score to be used in Buchholz for calculating one's own tie-break is always equal to the score obtained [16.4], so we need only calculate the adjusted scores to be used for opponents' tie-breaks (column AS-O). The techniques are the same used in previous exercises, so we will not go into detail. At this stage, it is most convenient to use the crosstable sorted by pairing numbers.

#	NAME	ELO	SCORE	1	2	3	4	5	AS-O
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	3.5
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	4.0
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	3.5
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	3.5

5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	2.5
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	3.0
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	1.5
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	2.5
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	1.5
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	1.0
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	2.5
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	3.0
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	1.5
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	2.0
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	2.0
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	3.5

To avoid trivial errors, let's remember that when calculating the tie-break of a player who has unplayed rounds, we must choose the right contribution between the face value (for themselves) and the adjusted one (for opponents, i.e., everyone else). Once all contributions have been determined, we can proceed to their sum, player by player, as in the following table (which has been sorted by score and Buchholz for future convenience).

#	NAME	ELO	SCORE	1	2	3	4	5	BH
2	Bruno	2150	4.0	1.0	1.5	3.5	3.5	3.5	13.0
3	Charline	2100	3.5	2.5	3.0	2.5	3.5	4.0	15.5
4	David	2050	3.5	3.0	3.5	1.5	3.5	3.5	15.0
1	Alyx	2200	3.5	1.5	1.5	4.0	2.0	3.5	12.5
16	Stephan	1450	3.5	2.5	2.5	1.5	4.0	2.0	12.5
6	Franck	1950	3.0	2.0	3.5	3.0	1.0	2.5	12.0
8	Irina	1850	2.5	3.5	2.0	3.5	1.5	3.0	13.5
11	Maria	1700	2.5	3.5	3.5	2.5	2.5	1.5	13.5
5	Helene	2000	2.5	1.5	2.0	2.5	1.5	1.0	8.5
15	Reine	1500	2.0	1.5	2.5	1.0	3.5	3.5	12.0
12	Nick (W)	1650	2.0	3.5	2.0	2.0	2.0	2.0	11.5
14	Paul	1550	2.0	3.0	2.5	2.0	2.0	1.5	11.0
7	Genevieve	1900	1.5	2.0	4.0	3.5	2.5	2.5	14.5
13	Opal	1600	1.5	2.5	3.5	3.5	2.5	2.0	14.0
9	Jessica	1800	1.5	3.5	1.0	1.5	1.5	1.5	9.0
10	Lais	1750	1.0	4.0	1.5	2.0	3.0	2.5	13.0

Now we calculate the average of the opponents' Buchholz values, keeping in mind that we should add only those opponents who were actually met on-the-board [8.2]. To avoid false ties caused by number rounding, we calculate the averages to two decimal places. Finally, we sort the list to get the ranking.

#	NAME	ELO	SCORE	1	2	3	4	5	BH	AOB
2	Bruno	2150	4.0	13.0	14.5	12.5	12.5	15.5	13.0	13.60
3	Charline	2100	3.5	13.5	12.0	13.5	15.0	13.0	15.5	13.40
4	David	2050	3.5	11.5	--	14.0	15.5	12.5	15.0	13.38
16	Stephan	1450	3.5	13.5	13.5	14.5	13.0	12.0	12.5	13.30
1	Alyx	2200	3.5	9.0	14.0	13.0	12.0	15.0	12.5	12.60
6	Franck	1950	3.0	11.0	15.5	--	13.0	13.5	12.0	13.25
5	Helene	2000	2.5	14.0	12.0	13.5	14.5	13.0	8.5	13.40
8	Irina	1850	2.5	12.5	11.0	15.5	14.0	12.0	13.5	13.00
11	Maria	1700	2.5	15.5	12.5	8.5	--	14.5	13.5	12.75
12	Nick (W)	1650	2.0	15.0	--	--	--	--	11.5	15.00
14	Paul	1550	2.0	12.0	13.5	--	--	14.0	11.0	13.17
15	Reine	1500	2.0	14.5	8.5	13.0	12.5	12.5	12.0	12.20
9	Jessica	1800	1.5	12.5	13.0	--	--	--	9.0	12.75
13	Opal	1600	1.5	8.5	12.5	15.0	13.5	11.0	14.0	12.10
7	Genevieve	1900	1.5	12.0	13.0	12.5	8.5	13.5	14.5	11.90
10	Lais	1750	1.0	13.0	9.0	12.0	12.0	8.5	13.0	10.90

3.4 Fore Buchholz

The essential feature of this tie-break is that it (as well as, for example, ARO) can be calculated as soon as the pairing of the last round is known, even before it is actually played. Modifiers can be applied to this tiebreaker, but here we will limit ourselves to an example of a total Fore Buchholz.

Exercise 10

In the Swiss tournament, using the Fore Buchholz (FB) system, determine the tie-break values and ranking order for all players.

The key feature of the system is all the last round games are considered as drawn (they may not have been played yet). Of course, *planned* unplayed games, being known *before* the round, will be considered as in reality – there is no need to make assumptions about their conclusion!

Now, by definition, the "final" score used for the tie-break calculation is not the real one. We therefore highlighted (in red) the scores that differ from the real results – and (of course) there are several of them. The score adjusted for opponents can also be different, also because the definition itself of this tie-break "conceals" any forfeits in the last round.

#	NAME	ELO	SCORE	1	2	3	4	5	AS-O
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	4.0
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	3.5
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	3.5
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	3.5
16	Stephan	1450	3.0	=W8	+B11	+W7	-B2	=W15	3.0
6	Franck	1950	2.5	-B14	-W3	+BYE	+W10	=B8	2.5
5	Helene	2000	2.0	-W13	-B15	+W11	=B7	=W10	2.0
8	Irina	1850	3.0	=B16	+W14	-B3	+W13	=W6	3.0
11	Maria	1700	2.0	=B3	-W16	-B5	+F9	=W7	2.0
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	3.0
14	Paul	1550	1.5	+W6	-B8	-F12	--	=B13	1.5
15	Reine	1500	2.5	-B7	+W5	+B10	-W1	=B16	2.5
7	Genevieve	1900	2.0	+W15	-B2	-B16	=W5	=B11	2.0
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	1.5
13	Opal	1600	2.0	+B5	=W1	-B4	-B8	=W14	2.0
10	Lais	1750	1.5	-W2	+B9	-W15	-B6	=B5	1.5

Having determined the contributions of each player for their opponents, we now proceed to calculate the tie-break values, using the same method as in the previous exercise. Rearranging then our crosstable based on the tie-break values, we obtain:

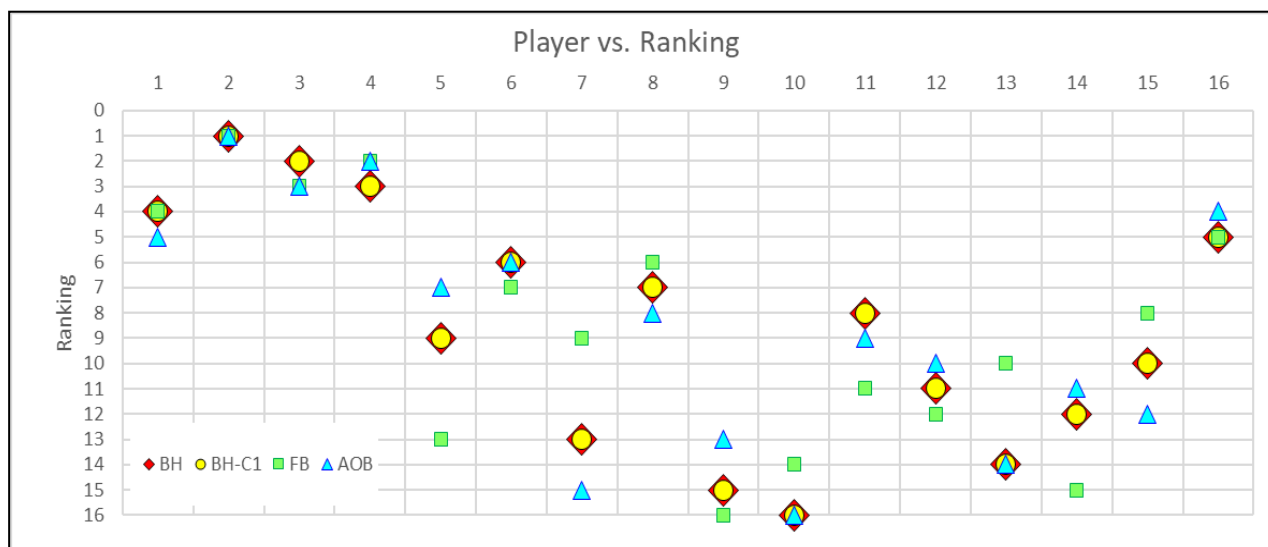
#	NAME	ELO	SCORE	1	2	3	4	5	FB
2	Bruno	2150	4.0	1.5	2.0	3.5	3.0	3.5	13.5
3	Charline	2100	3.5	2.0	2.5	3.0	3.5	4.0	15.0
4	David	2050	3.5	3.0	3.5	2.0	3.5	3.5	15.5
1	Alyx	2200	3.5	1.5	2.0	4.0	2.5	3.5	13.5
16	Stephan	1450	3.0	3.0	2.0	2.0	4.0	2.5	13.5
8	Irina	1850	3.0	3.0	1.5	3.5	2.0	2.5	12.5
6	Franck	1950	2.5	1.5	3.5	2.5	1.5	3.0	12.0
15	Reine	1500	2.5	2.0	2.0	1.5	3.5	3.0	12.0
5	Helene	2000	2.0	2.0	2.5	2.0	2.0	1.5	10.0
11	Maria	1700	2.0	3.5	3.0	2.0	2.0	2.0	12.5
12	Nick (W)	1650	2.0	3.5	2.0	2.0	2.0	2.0	11.5
14	Paul	1550	1.5	2.5	3.0	1.5	1.5	2.0	10.5
7	Genevieve	1900	2.0	2.5	4.0	3.0	2.0	2.0	13.5
9	Jessica	1800	1.5	3.5	1.5	1.5	1.5	1.5	9.5
13	Opal	1600	2.0	2.0	3.5	3.5	3.0	1.5	13.5
10	Lais	1750	1.5	4.0	1.5	2.5	2.5	2.0	12.5

Let's close this chapter with an interesting comparison of the rankings produced by the various systems; the following table shows the ranking of each player, identified by their pairing number. Unresolved ties are highlighted. These instances require further tie-breakers, or the drawing of lots, to determine final standings.

	Final Rankings															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
BH	2	3	4	1	16	6	8	11	5	15	12	14	7	13	9	10
BH-C1	2	3	4	1	16	6	8	11	5	15	12	14	7	13	9	10
AOB	2	4	3	16	1	6	5	8	11	12	14	15	9	13	7	10
FB	2	4	3	1	16	8	6	15	7	13	11	12	5	10	14	9

We can see that the generated rankings vary, potentially influencing the allocation of titles and prizes. Moreover, none of these systems has successfully resolved all ties.

The diagram below illustrates the variations in the final ranking position for each player (horizontal axis), determined by the tie-break system used.



4 TIE-BREAKS WITH SONNEBORN-BERGER AND KOYA SYSTEMS

Contrary to Buchholz, the Sonneborn-Berger (SB) system can be used both for Swiss tournaments and for round-robins, although it is not the most suitable option for the former. For Sonneborn-Berger, the same modifiers applicable to the Buchholz can be used (especially the Cut-1, while it makes no sense to use the Median), but the use of variants of the SB with modifiers is rare.

On the other hand, the Koya system is specifically defined for round-robin tournaments. If an organizer intends to use it in a Swiss tournament, it must be explicitly defined in the event regulations [4.1]). The Koya system allows for the use of the Limit modifier.

4.1 Sonneborn-Berger for Swiss tournaments

In the Sonneborn-Berger system [9.1], the results of opponents are combined with those of the player in such a way that the opponent's score has less weight in the case of a draw and is effectively ignored in the case of a loss. *Unlike Buchholz, a defeat against a strong player does not contribute in any way in this system.* On one hand, this system acknowledges that there is no special merit in losing to a Grandmaster; on the other hand, a player of average skill who happens to face several extremely strong players would be

penalized without any real fault of their own. As is often the case with tie-breaks, the interpretation of the meaning of the system used, and therefore its fairness, *is more of a philosophical than a technical matter*. However, in principle, the choice of tie-breaks is up to the event organizer, and the player can decide whether to participate after reviewing the rules.

We will now see some examples similar to those used for Buchholz, so we will be able to compare the results and get an idea of the different behaviour between the two systems.

Exercise 11

In the Swiss tournament, using the Sonneborn-Berger (SB) system, determine the tie-break values and ranking order for the players at 3.5 points.

Affected players are #1, #3, #4, #16. The management rules for unplayed rounds are the same as in the case of Buchholz; in SB, however, for correct application, it is essential to clearly understand the difference between the opponent's value, which is their score, and their contribution to the tie-break [9.1].

The opponent's contribution is each addend that must be added to obtain the value of the tie-break. It is given by the product between the opponent's score and the result obtained against them (the contributions of the opponents with which the player lost are therefore all null). This should not be confused with the value itself.

Here, the only score that must be adjusted for the calculation of the opponents' tie-break is that of player #12, who withdrew from the fourth round on (see Exercise 4).

As usual, let's examine the players and their opponents one by one.

#1	NAME	ELO	SCORE	1	2	3	4	5
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Player #1 played all of their games; furthermore, their opponents' unplayed rounds are all to be counted at face value [16.3.1]. To avoid errors, in the calculations we'd better follow the sequence of opponents as shown on the crosstable; this player's tiebreak is therefore $SB(\#1) = 1*1.5 + \frac{1}{2}*1.5 + \frac{1}{2}*4.0 + 1*2.0 + \frac{1}{2}*3.5 = 8.00$.

#3	NAME	ELO	SCORE	1	2	3	4	5
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7

Let's now consider player #3. Their opponents' unplayed rounds are all accounted for at face value, and therefore $SB(\#3) = \frac{1}{2} \cdot 2.5 + 1 \cdot 3.0 + 1 \cdot 2.5 + \frac{1}{2} \cdot 3.5 + \frac{1}{2} \cdot 4.0 = 10.50$.

#16	NAME	ELO	SCORE	1	2	3	4	5
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11

Going on to player #16, again nothing changes, and we find

$$SB(\#16) = \frac{1}{2} \cdot 2.5 + 1 \cdot 2.5 + 1 \cdot 1.5 + 0 \cdot 4.0 + 1 \cdot 2.0 = 7.25.$$

#4	NAME	ELO	SCORE	1	2	3	4	5
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14

Finally, player #4 got a half-point bye, and met the withdrawn player. The half-point bye on request provides them with a value equal to their score, i.e., 3.5 points, as per the crosstable. Let's be careful, however: to obtain the *contribute* of the (dummy) opponent, this value must be multiplied by the equivalent result of the round, which is a draw [16.4].

The contribution of the withdrawn opponent is calculated, as we saw above, by evaluating the unplayed rounds since the withdrawal as draws. It is therefore worth 3.0 points. So

$$SB(\#4) = 1 \cdot 3.0 + \frac{1}{2} \cdot 3.5 + 1 \cdot 1.5 + \frac{1}{2} \cdot 3.5 + \frac{1}{2} \cdot 3.5 = 9.75.$$

The ranking therefore is #3 (10.50), #4 (9.75), #1 (8.00) and #16 (7.25).

Exercise 12

In the Swiss tournament, using the Sonneborn-Berger (SB) system, determine the tie-break values and ranking order for all players.

For each player, we calculate the score adjusted for opponents' tie-break, and put it in the "AS" column we added to the crosstable. Then we can calculate the contribution given by each opponent's score – for clarity, we explicitly show the product between the player's result and the opponent's score. The last column of the table contains the sum of all the contributions, i.e., the tie-break value; all that remains to do, is to sort players to yield the final ranking.

Methodological note: For this exercise and the following ones, the reader is strongly encouraged to start from the table presented in paragraph 2.1, perform the calculation independently, and only then compare it with the one reported here to verify its accuracy.

#	NAME	SCORE	AS	1	2	3	4	5	SB
2	Bruno	4.0	4.0	+B10	+W7	=B1	+W16	=B3	9.50
				1*1.0	1*1.5	½*3.5	1*3.5	½*3.5	
3	Charline	3.5	3.5	=W11	+B6	+W8	=B4	=W2	10.50
				½*2.5	1*3.0	1*2.5	½*3.5	½*4.0	
4	David	3.5	3.5	+B12	=BYE	+W13	=W3	=B1	9.75
				1*3.0	½*3.5	1*1.5	½*3.5	½*3.5	
1	Alyx	3.5	3.5	+W9	=B13	=W2	+B15	=W4	8.00
				1*1.5	½*1.5	½*4.0	1*2.0	½*3.5	
16	Stephan	3.5	3.5	=W8	+B11	+W7	-B2	+W15	7.25
				½*2.5	1*2.5	1*1.5	0*4.0	1*2.0	
6	Franck	3.0	3.0	-B14	-W3	=BYE	+W10	+B8	6.50
				0*2.0	0*3.5	1*3.0	1*1.0	1*2.5	
11	Maria	2.5	2.5	=B3	-W16	-B5	+F9	+W7	5.75
				½*3.5	0*3.5	0*2.5	1*2.5	1*1.5	
8	Irina	2.5	2.5	=B16	+W14	-B3	+W13	-W6	5.25
				½*3.5	1*2.0	0*3.5	1*1.5	0*3.0	
5	Helene	2.5	2.5	-W13	-B15	+W11	=B7	+W10	4.25
				0*1.5	0*2.0	1*2.5	½*1.5	1*1.0	
14	Paul	2.0	2.0	+W6	-B8	-F12	--	+B13	4.50
				1*3.0	0*2.5	0*2.0	0*2.0	1*1.5	
12	Nick (W)	2.0	3.0	-W4	=BYE	+F14	--	--	4.00
				0*3.5	1*2.0	1*2.0	0*2.0	0*2.0	
15	Reine	2.0	2.0	-B7	+W5	+B10	-W1	-B16	3.50
				0*1.5	1*2.5	1*1.0	0*3.5	0*3.5	
13	Opal	1.5	1.5	+B5	=W1	-B4	-B8	-W14	4.25
				1*2.5	½*3.5	0*3.5	0*2.5	0*2.0	
7	Genevieve	1.5	1.5	+W15	-B2	-B16	=W5	-B11	3.25
				1*2.0	0*4.0	0*3.5	½*2.5	0*2.5	
9	Jessica	1.5	1.5	-B1	-W10	=BYE	-F11	+BYE	2.25
				0*3.5	0*1.0	½*1.5	0*1.5	1*1.5	
10	Lais	1.0	1.0	-W2	+B9	-W15	-B6	-B5	1.50
				0*4.0	1*1.5	0*2.0	0*3.0	0*2.5	

Exercise 13

In the Swiss tournament, using the Sonneborn-Berger Cut-1 (SB-C1) system, determine the tie-break values and ranking order for all players.

The application of the Cut-1 modifier [14.1] to the Sonneborn-Berger system is, in practice, a novelty, but it may possibly become more widespread over time.

The modifier requires disregarding, among all opponent contributions, the least significant one and, once again, this corresponds to the opponent with the lowest score [14.1.1].

Now, however, since the opponent's contribution, i.e., the value to be actually added, also depends on the player's result, the least significant value may not be the one that gives rise to the smallest contribution. For instance, a draw against a three-point opponent is worth 1.5, which is smaller than the contribution of a win against a two-point opponent (2.0); however, the least significant value is the latter, and *this is the one to cut*.

If two or more opponents share the minimum score, the opponent that gives the smallest contribution to the player will be cut. For example, if both "less significant" opponents have two points, and the player has won against one while losing or drawing against the other, the latter will be cut.

When the player has one or more unplayed rounds due to their unavailability (forfeit losses; requested byes; scheduled absences) [16.1.2], the Cut-1 instead cuts one of these (two in the case of Cut-2, and so on) – but only if the corresponding contribution is not less than the least significant value [16.5]; in this latter case, the exception does not apply, and the opponent with the least significant value is cut.

Note: among voluntarily unplayed rounds (VUR), only half-point byes (HPB) bring a non-zero contribution, while forfeits and zero-point byes always give a null contribution. The round to be cut is determined by finding (1) the smallest contribution due to a VUR and (2) the contribution due to the least significant opponent (whether real or dummy), which is always the one with the lowest score. The largest between these two is the one to be discarded.

As in the previous exercise, let's calculate all the opponents' values for each player; this time however, before performing the sum, we need to cut a value from the calculation (in the ensuing table, the cut values relating to played games are highlighted in blue, while those relating to unplayed rounds are in red).

#	NAME	SCORE	AS	1	2	3	4	5	SB-C1
2	Bruno	4.0	4.0	+B10	+W7	=B1	+W16	=B3	8.50
				1*1.0	1*1.5	½*3.5	1*3.5	½*3.5	
3	Charline	3.5	3.5	=W11	+B6	+W8	=B4	=W2	9.25
				½*2.5	1*3.0	1*2.5	½*3.5	½*4.0	
4	David	3.5	3.5	+B12	=BYE	+W13	=W3	=B1	8.00
				1*3.0	½*3.5	1*1.5	½*3.5	½*3.5	
1	Alyx	3.5	3.5	+W9	=B13	=W2	+B15	=W4	7.25
				1*1.5	½*1.5	½*4.0	1*2.0	½*3.5	
16	Stephan	3.5	3.5	=W8	+B11	+W7	-B2	+W15	5.75
				½*2.5	1*2.5	1*1.5	0*4.0	1*2.0	
6	Franck	3.0	3.0	-B14	-W3	+BYE	+W10	+B8	5.50
				0*2.0	0*3.5	1*3.0	1*1.0	1*2.5	
11	Maria	2.5	2.5	=B3	-W16	-B5	+F9	+W7	4.25
				½*3.5	0*3.5	0*2.5	1*2.5	1*1.5	
8	Irina	2.5	2.5	=B16	+W14	-B3	+W13	-W6	3.75
				½*3.5	1*2.0	0*3.5	1*1.5	0*3.0	
5	Helene	2.5	2.5	-W13	-B15	+W11	=B7	+W10	3.25
				0*1.5	0*2.0	1*2.5	½*1.5	1*1.0	
12	Nick (W)	2.0	3.0	-W4	+BYE	+F14	--	--	4.00
				0*3.5	1*2.0	1*2.0	0*2.0	0*2.0	
14	Paul	2.0	2.0	+W6	-B8	-F12	--	+B13	3.00
				1*3.0	0*2.5	0*2.0	0*2.0	1*1.5	
15	Reine	2.0	2.0	-B7	+W5	+B10	-W1	-B16	2.50
				0*1.5	1*2.5	1*1.0	0*3.5	0*3.5	
13	Opal	1.5	1.5	+B5	=W1	-B4	-B8	-W14	4.25
				1*2.5	½*3.5	0*3.5	0*2.5	0*2.0	

7	Genevieve	1.5	1.5	+W15	-B2	-B16	=W5	-B11	1.25
				1*2.0	0*4.0	0*3.5	½*2.5	0*2.5	
9	Jessica	1.5	1.5	-B1	-W10	=BYE	-F11	+BYE	2.25
				0*3.5	0*1.0	½*1.5	0*1.5	1*1.5	
10	Lais	1.0	1.0	-W2	+B9	-W15	-B6	-B5	0.00
				0*4.0	1*1.5	0*2.0	0*3.0	0*2.5	

To clarify better, let's see the choice of the value to discard for some players:

- #2 the least significant contribution, and therefore the one to be discarded, is due to opponent #10, who obtained the lowest score
- #4 we have a contribution of 1.75 points due to an HPB, while the least significant among those due to the opponents (precisely, at #13) is worth 1.50 points. The larger of the two is discarded, which is the one relating to the HPB
- #6 the least significant contribution, the one to be discarded, is due to opponent #10, with whom the player won; the minimum contribution instead would be the one due to #14, but *this opponent is not the least significant and should not be discarded*
- #14 here there is a contribution due to the forfeit or scheduled absence (it makes no difference, they are both null); the least significant value is that due to opponent #13 and the relative contribution is worth 1.50 points. We discard the larger value, which in this case is the one due to the *actually played game*
- #12 there is a zero contribution due to a scheduled absence (fourth or fifth round, it makes no difference which of the two – please note that the PAB and the forfeit win are not rounds without availability to play). The least significant are still those due to these unplayed rounds because the dummy opponent has the lowest score. The round to be discarded is therefore one of these two, and its contribution is zero
- #9 between the two rounds without availability to play (VUR), the minimum contribution is zero due to the forfeit defeat (the HPB instead gives a contribution of 0.75 points); the one relating to the least significant value is due to opponent #10, and is also worth zero, so the first one is discarded

4.2 Sonneborn-Berger in round-robin tournaments

In round-robins, which are tournaments with predetermined pairings, unplayed games can only be won or lost by forfeit. They are treated on a par with regularly played games [15.6], so it is never necessary to adjust the scores obtained. Apart from this, there is no substantial difference as compared to what we did in the case of the Swiss tournament.

Exercise 14

In the Round-robin tournament determine the tie-break values and ranking order for all players using the Sonneborn-Berger (SB) system.

As we did before, we insert the contributions of the individual matches into the crosstable; note that the 5-6 game, awarded by forfeit, is treated just like all other games.

#	NAME	ELO	SCORE	1	2	3	4	5	SB
1	Alyx	2200	3.5	=W5	+W2	+B3	-W4	+B6	9.25
				½*1.5	1*3.5	1*3.5	0*1.5	1*1.5	
2	Bruno	2150	3.5	+W6	-B1	+W5	=W3	+B4	6.25
				1*1.5	0*3.5	1*1.5	½*3.5	1*1.5	
3	Charline	2100	3.5	+W4	+B6	-W1	=B2	+W5	6.25
				1*1.5	1*1.5	0*3.5	½*3.5	1*1.5	
4	David	2050	1.5	-B3	-B5	=W6	+B1	-W2	4.25
				0*3.5	0*1.5	½*1.5	1*3.5	0*3.5	
5	Franck	1950	1.5	=B1	+W4	-B2	-W6	-B3	3.25
				½*3.5	1*1.5	0*3.5	0*1.5	0*3.5	
6	Helene	2000	1.5	-B2	-W3	=B4	+B5	-W1	2.25
				0*3.5	0*3.5	½*1.5	1*1.5	0*3.5	

Exercise 15

In the Round-robin tournament determine the tie-break values and ranking order for all players using the Sonneborn-Berger Cut-1 (SB-C1) system.

As mentioned, here we do not need to apply adjustments to unplayed games. Since we use the Cut-1 modifier, for each player we will cut the contribution of the least significant opponent (the one with the lowest score) – and, when scores are equal, we cut the lower of those contributions [14.1.1] (the cut round is highlighted in blue on the crosstable).

#	NAME	ELO	SCORE	1	2	3	4	5	SB-C1
1	Alyx	2200	3.5	=W5	+W2	+B3	-W4	+B6	9.25
				½*1.5	1*3.5	1*3.5	0*1.5	1*1.5	
2	Bruno	2150	3.5	+W6	-B1	+W5	=W3	+B4	4.75
				1*1.5	0*3.5	1*1.5	½*3.5	1*1.5	
3	Charline	2100	3.5	+W4	+B6	-W1	=B2	+W5	4.75
				1*1.5	1*1.5	0*3.5	½*3.5	1*1.5	
4	David	2050	1.5	-B3	-B5	=W6	+B1	-W2	4.25
				0*3.5	0*1.5	½*1.5	1*3.5	0*3.5	
5	Franck	1950	1.5	=B1	+W4	-B2	-W6	-B3	3.25
				½*3.5	1*1.5	0*3.5	0*1.5	0*3.5	
6	Helene	2000	1.5	-B2	-W3	=B4	+B5	-W1	1.50
				0*3.5	0*3.5	½*1.5	1*1.5	0*3.5	

(Please note that the ranking order is the same obtained without Cut 1 just by a coincidence.)

4.3 Koya System

The Koya system is defined only for round-robin tournaments. This does not mean that its use in a Swiss tournament is forbidden – but Organisers who intend to use it must define it in their tournament rules ([4.1]). An Arbiter however cannot introduce it as an additional tie-break because they can choose only among the methods listed in [5].

Exercise 16

In the round-robin tournament, determine the tie-break values and the rankings for all players using the Koya system (KS). In case of persistent ties, apply the Limit modifier to each group of tied players until achieving, if possible, a unique ranking.

In the Koya system, we sum the points obtained by each player against opponents who scored at least half of the maximum possible [9.2] – in our case, $5/2=2.5$ points. For clarity, we record all contributions due to players with a sufficient score along with the achieved result, while we marked the results against opponents with a score below the required minimum with a "—".

#	NAME	ELO	SCORE	1	2	3	4	5	KS
1	Alyx	2200	3.5	=W5	+W2	+B3	-W4	+B6	2.0
				—	1	1	—	—	
2	Bruno	2150	3.5	+W6	-B1	+W5	=W3	+B4	0.5
				—	0	—	½	—	
3	Charline	2100	3.5	+W4	+B6	-W1	=B2	+W5	0.5
				—	—	0	½	—	
4	David	2050	1.5	-B3	-B5	=W6	+B1	-W2	1.0
				0	—	—	1	0	
5	Franck	1950	1.5	=B1	+W4	-B2	-W6	-B3	0.5
				½	—	0	—	0	
6	Helene	2000	1.5	-B2	-W3	=B4	+B5	-W1	0.0
				0	0	—	—	0	

The tie between players #2 and #3 persists, so we will try to apply the Limit modifier [14.5] to these two players. Since the players are divided into only two groups with the same score, increasing the score limit by half a point at a time is useless, as at some point the contributions will simply be all ignored. By decreasing the limit score by half a point at a time, however, we will reach the point where all the results obtained will be considered, so the sum of the contributions becomes equal to the score. In conclusion, with the Koya system it is not possible to break this tie; we will have to resort to the next tie-break method provided for by the tournament regulations, or to the drawing of sorts [4.2].

At the end of the chapter, let's compare the outcomes in the Swiss tournament obtained using the tie-break systems examined thus far, which are commonly employed in these competitions. The following table, sorted according to the Buchholz system, illustrates the player's ranking given by each tie-break method, with different colours denoting positions to help a quick visual comparison of the different systems (matching colours show unsettled ties).

Sonneborn-Berger systems yield fewer persisting ties than systems relying on Buchholz. This seems to show a better discriminatory capacity, although further evidence is needed. The ability to distinctly differentiate between tied players is of course a positive attribute of a tie-break system - indeed, it is its primary purpose. However, discrimination alone is not enough – it must also be done *well*, meaning that the resulting ranking should ideally

reflect the players' playing strength expressed in the tournament. Yet, this discussion delves into a very complex issue that extends far beyond our objectives.

RNK	#	NAME	ELO	SCORE	BH	BH-C1	AOB	FB	SB	SB-C1
1	2	Bruno	2150	4,0	13,0	12,0	13,6	13,5	9,50	8,50
2	3	Charline	2100	3,5	15,5	13,0	13,4	15,0	10,50	9,25
3	4	David	2050	3,5	15,0	11,5	13,4	15,5	9,75	8,00
4	1	Alyx	2200	3,5	12,5	11,0	12,6	13,5	8,00	7,25
5	16	Stephan	1450	3,5	12,5	11,0	13,3	13,5	7,25	5,75
6	6	Franck	1950	3,0	12,0	11,0	13,3	12,0	6,50	5,50
7	8	Irina	1850	2,5	13,5	12,0	13,0	12,5	5,25	3,75
8	11	Maria	1700	2,5	13,5	12,0	12,8	12,5	5,75	4,25
9	5	Helene	2000	2,5	8,5	7,5	13,4	10,0	4,25	3,25
10	15	Reine	1500	2,0	12,0	11,0	12,2	12,0	3,50	2,50
11	12	Nick (W)	1650	2,0	11,5	9,5	15,0	11,5	4,00	4,00
12	14	Paul	1550	2,0	11,0	9,0	13,2	10,5	4,50	3,00
13	7	Genevieve	1900	1,5	14,5	12,5	11,9	13,5	3,25	1,25
14	13	Opal	1600	1,5	14,0	12,0	12,1	13,5	4,25	4,25
15	9	Jessica	1800	1,5	9,0	7,5	12,8	9,5	2,25	2,25
16	10	Lais	1750	1,0	13,0	11,5	10,9	12,5	1,50	0,00

Key:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
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5 TIE-BREAK SYSTEMS BASED ON RATING AND PERFORMANCE

Whenever unrated players are involved, these tie-breaks are undefined and *should therefore be removed from the tie-break list*. However, Organisers can use them if they wish so, provided that they clearly define the rules for the management of such players in the tournament regulations. Alternatively, the chief arbiter of the tournament can establish these rules and make them public *before the tournament begins*. In either case, those rules must be:

- Impartial (based solely on the chess-related characteristics of the player)
- Fair (should not personally favour or disadvantage any player)
- Comprehensive (must handle all possible cases)
- Unambiguous (should leave no doubts about application in every possible scenario)

All tie-breaks in this category are based solely on games *actually played* on-the-board, while unplayed games are ignored. For example, for a player who missed two games in a nine-round tournament, only the seven valid results are considered, and the average is calculated by dividing by seven (not nine!).

When ratings have decimals (e.g., in averages), the result is always rounded to the nearest whole number, following the convention 4/5 (3.4 → 3, but 3.5 → 4).

5.1 Average Rating of opponents (ARO)

This is the average of all the ratings of the opponents who were actually met on-the-board [10.1]; When ratings are reliable, we can assume this average to be a good estimate of the strength of the encountered opposition –the underlying idea being that the same score achieved against stronger opposition carries greater merit.

This tie-break is independent of the achieved results and can therefore be calculated even while the round is still in progress – the only required information is that the game has actually taken place. In a tournament with predetermined pairings, such as a round-robin, where unplayed games are treated the same as played ones, this tie-break can even be calculated at the beginning of the tournament (and favours lower rated players).

For this tie-break, unique in its category, the Cut modifier can be applied.

Exercise 17

In the Swiss tournament, using the ARO system, determine the tie-break values and ranking order for all players.

As usual, we start from the crosstable, adding data as needed – here, we need all the opponents' ratings. The average is calculated adding up all valid contributions and then dividing by their number – while all unplayed rounds are completely ignored.

As an example, let's see in detail the calculations for player #4. Their opponents are #12 (rating 1650), #13 (1600), #3 (2100) and #1 (2200). The unplayed game in the second round is discarded.

The average of ratings (ARO) is given by the sum of opponents rating divided by their number: $ARO = (1650+1600+2100+2200)/4$. Hence, $ARO = 7550/4 = 1887,5 \rightarrow 1888$. The calculation of the ARO for all other players, which is carried out using the same procedure, is left as an exercise for the reader.

#	NAME	ELO	SCORE	1	2	3	4	5	ARO
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	1880
				1750	1900	2200	1450	2100	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	1940
				1700	1950	1850	2050	2150	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	1888
				1650	--	1600	2100	2200	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	1820
				1800	1600	2150	1500	2050	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	1820
				1850	1700	1900	2150	1500	
6	Franck	1950	3.0	-B14	-W3	=BYE	+W10	+B8	1813
				1550	2100	--	1750	1850	
11	Maria	1700	2.5	=B3	-W16	-B5	=F9	+W7	1863
				2100	1450	2000	--	1900	
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	1730
				1450	1550	2100	1600	1950	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	1690
				1600	1500	1700	1900	1750	

12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	2050
				2050	--	--	--	--	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	1860
				1900	2000	1750	2200	1450	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	1800
				1950	1850	--	--	1600	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	1975
				2200	1750	--	--	--	
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	1930
				2000	2200	2050	1850	1550	
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	1760
				1500	2150	1450	2000	1700	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	1880
				2150	1800	1500	1950	2000	

Exercise 18

In the Swiss tournament, using the ARO Cut-1 system (AROC), determine the tie-break values and ranking order for all players.

The only difference from the previous exercise is the choice of the contribution to be cut, which is always the one related to the least significant result. In the case of AROC, which is based on ratings, the least significant encounter is that with the lowest rated opponent. This contribution must therefore be the one to be discarded before calculating the average.

#	NAME	ELO	SCORE	1	2	3	4	5	AROC
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	1988
				1750	1900	2200	1450	2100	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	2000
				1700	1950	1850	2050	2150	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	1983
				1650	--	1600	2100	2200	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	1900
				1800	1600	2150	1500	2050	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	1900
				1850	1700	1900	2150	1500	
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	1900
				1550	2100	--	1750	1850	
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	2000
				2100	1450	2000	--	1900	
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	1800
				1450	1550	2100	1600	1950	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	1738
				1600	1500	1700	1900	1750	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	1963
				1900	2000	1750	2200	1450	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	1900
				1950	1850	--	--	1600	
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	0
				2050	--	--	--	--	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	2200
				2200	1750	--	--	--	
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	2025
				2000	2200	2050	1850	1550	

7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	1838
				1500	2150	1450	2000	1700	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	1975
				2150	1800	1500	1950	2000	

For player #12, who played only one game, after the cut there is nothing left to calculate a tie-break value. Hence, their ARO is *undefined*. Incidentally, there is no definite rule to handle this case.

5.2 Performance rating in the tournament (TPR)

The aim of this tie-break is to estimate (approximately) how well a competitor has played based on their results, in relation to the opponents' playing strength as represented by their ratings.

When two players face each other, the expected score (i.e., the statistical probability of winning) is determined by the difference between their ratings, as per the table in section 8.1.1 of the *FIDE Rating Regulations* (FIDE Handbook B.02). If a player faced several opponents and achieved a given average score, the table provides the corresponding rating difference for that average score. The performance value is obtained by adding this rating difference to the player's ARO, and it is an approximate estimate of the rating that the player would have needed to achieve those results against that opposition (for more details see the in-depth box on the next page).

Note: if two players obtained equal *over-the-board* scores, the rating difference is of course the same. The ranking yielded by this method is therefore the very same given by ARO. However, this is not always the case, because the same final score can be obtained with different contributions by unplayed rounds, which the tie-breaker disregards.

What is the meaning of TPR?

To clarify the meaning of TPR, let's calculate the expected score of a player, say #2, assuming a rating equal to the performance. To do that, we need to find the rating differences with respect to each opponent met, then we proceed to find the expected score for each game using the table in B.02-8.1.2 (see below), which is the mirror image of the table mentioned above.

d			dp			d			dp			d			dp				
Rtg Dif			H	L	Rtg Dif			H	L	Rtg Dif			H	L	Rtg Dif			H	L
0-3			.50	.50	92-98			.63	.37	198-206			.76	.24	345-357			.89	.11
4-10			.51	.49	99-106			.64	.36	207-215			.77	.23	358-374			.90	.10
11-17			.52	.48	107-113			.65	.35	216-225			.78	.22	375-391			.91	.09
18-25			.53	.47	114-121			.66	.34	226-235			.79	.21	392-411			.92	.08
26-32			.54	.46	122-129			.67	.33	236-245			.80	.20	412-432			.93	.07
33-39			.55	.45	130-137			.68	.32	246-256			.81	.19	433-456			.94	.06
40-46			.56	.44	138-145			.69	.31	257-267			.82	.18	457-484			.95	.05
47-53			.57	.43	146-153			.70	.30	268-278			.83	.17	485-517			.96	.04
54-61			.58	.42	154-162			.71	.29	279-290			.84	.16	518-559			.97	.03
62-68			.59	.41	163-170			.72	.28	291-302			.85	.15	560-619			.98	.02
69-76			.60	.40	171-179			.73	.27	303-315			.86	.14	620-735			.99	.01
77-83			.61	.39	180-188			.74	.26	316-328			.87	.13	> 735			1.0	.00
84-91			.62	.38	189-197			.75	.25	329-344			.88	.12					

Using the performance value (2120) instead of the rating (2150) and remembering that the rules limit the rating differences to ± 400 points (see B.02-8.3.1), we find:

Opp.	Rating	Rating difference	dp
10	1750	2120 - 1750 = +370	0.90
7	1900	2120 - 1900 = +220	0.78
1	2200	2120 - 2200 = -80	0.39
16	1450	2120 - 1450 = +670 \rightarrow +400	0.92
3	2100	2120 - 2100 = +20	0.53

The expected score P_a is the sum of the winning probabilities, i.e.,

$$P_a = 0.90 + 0.78 + 0.39 + 0.92 + 0.53 = 3.52$$

This value approximates the real score, but indeed the approximation is not a brilliant one. Had we calculated the expected score by means of the averaged ratings (ARO), as it was prescribed in past rules, instead of round by round as is the rule today, we would have obtained a more precise result:

TPR-ARO = 2120-1880 = 240, hence $P_d = 0.80$, and $P_a = 5 \times 0.80 = 4.0$ (but the results are not usually *that* precise).

In summary, *TPR is an approximation (and not a very precise one...) of the rating the player should have in order to justify their result* (we will see presently that PTP is a better estimate for this value).

Exercise 19

In the Swiss tournament, calculate the TPR of players #2, #6 and #12.

ARO is of course calculated as we did in the previous exercise. For convenience, let's reproduce the table from B.02-8.1.1

p	dp	p	dp	p	dp	p	dp	p	dp	p	dp
1.0	800	.83	273	.66	117	.49	-7	.32	-133	.15	-296
.99	677	.82	262	.65	110	.48	-14	.31	-141	.14	-309
.98	589	.81	251	.64	102	.47	-21	.30	-149	.13	-322
.97	538	.80	240	.63	95	.46	-29	.29	-158	.12	-336
.96	501	.79	230	.62	87	.45	-36	.28	-166	.11	-351
.95	470	.78	220	.61	80	.44	-43	.27	-175	.10	-366
.94	444	.77	211	.60	72	.43	-50	.26	-184	.09	-383
.93	422	.76	202	.59	65	.42	-57	.25	-193	.08	-401
.92	401	.75	193	.58	57	.41	-65	.24	-202	.07	-422
.91	383	.74	184	.57	50	.40	-72	.23	-211	.06	-444
.90	366	.73	175	.56	43	.39	-80	.22	-220	.05	-470
.89	351	.72	166	.55	36	.38	-87	.21	-230	.04	-501
.88	336	.71	158	.54	29	.37	-95	.20	-240	.03	-538
.87	322	.70	149	.53	21	.36	-102	.19	-251	.02	-589
.86	309	.69	141	.52	14	.35	-110	.18	-262	.01	-677
.85	296	.68	133	.51	7	.34	-117	.17	-273	.00	-800
.84	284	.67	125	.50	0	.33	-125	.16	-284		

Now let's consider Player #2 and calculate their ARO.

#	NAME	ELO	SCORE	1	2	3	4	5	ARO
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	1880
				1750	1900	2200	1450	2100	

Player #2 scored 4 points in five games, achieving an average score of $p = 4/5 = 0.8$. We look up this value in the table, finding that it corresponds to an expected rating difference $dp = 240$ points. Adding this difference to the player's ARO, we find

$$\text{TPR} = \text{ARO} + dp = 1880 + 240 = 2120.$$

Let's now proceed to player #6, whose unplayed game will be ignored (i.e., discarded):

#	NAME	ELO	SCORE	1	2	3	4	5	ARO
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	1813
				1550	2100	--	1750	1850	

Since there are only four played games, while the PAB must be ignored, we have $p = 2/4 = 0.50$, corresponding to a $dp = 0$, and hence $\text{TPR} = 1813 + 0 = 1813$.

Finally, let's consider player #12, who played only one game.

#	NAME	ELO	SCORE	1	2	3	4	5	ARO
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	2050
				2050	--	--	--	--	

The average score is zero. In this case, the table indicates a hypothetical rating difference of -800 points (which is not derived from a probability calculation but rather a technical choice). Therefore, we have $\text{TPR} = 2050 - 800 = 1250$.

Exercise 20

In the Swiss tournament, calculate TPR for all players.

The preliminary calculation of ARO is made just as in previous exercises, and we'll use the corresponding results. Since unplayed games must be ignored, we include the scores adjusted excluding the unrated games and the actual number N of played games.

#	NAME	ELO	ARO	SCORE	ADJ SC	N	p	dp	TPR
2	Bruno	2150	1880	4.0	4.0	5	0.80	240	2120
3	Charline	2100	1940	3.5	3.5	5	0.70	149	2089
4	David	2050	1888	3.5	3.0	4	0.88	193	2081
1	Alyx	2200	1820	3.5	3.5	5	0.70	149	1969
16	Stephan	1450	1820	3.5	3.5	5	0.70	149	1969
6	Franck	1950	1813	3.0	2.0	4	0.75	0	1813
11	Maria	1700	1863	2.5	1.5	4	0.63	-87	1776
8	Irina	1850	1730	2.5	2.5	5	0.50	0	1730
5	Helene	2000	1690	2.5	2.5	5	0.50	0	1690
14	Paul	1550	1800	2.0	2.0	3	0.67	125	1925
15	Reine	1500	1860	2.0	2.0	5	0.40	-72	1788
12	Nick (W)	1650	2050	2.0	0.0	1	2.00	-800	1250
13	Opal	1600	1930	1.5	1.5	5	0.30	-149	1781
7	Genevieve	1900	1760	1.5	1.5	5	0.30	-149	1611
9	Jessica	1800	1975	1.5	0.0	2	0.75	-800	1175
10	Lais	1750	1880	1.0	1.0	5	0.20	-240	1640

5.3 Average performance rating of opponents (APRO)

APRO is the average of the opponents' performances (TPR) rounded to the nearest whole number ([10.4]). The underlying idea is that if TPR approximates the playing strength expressed in the tournament by a player, the average of the opponents' TPRs approximates the average strength of the opposition actually faced. Consequently, the same result achieved against stronger opposition is considered more valuable. In other words, this tie-break emphasizes the quality of results against stronger opponents.

On the other hand, to calculate APRO, you need to compute the TPR of all opponents, making it a somewhat laborious tie-break.

Exercise 21

In the Swiss tournament, determine the ranking of 3.5-points players using APRO system.

Of course, the (preliminary) calculation of ARO is made just as in the above exercises, and we'll use the corresponding results. Since unplayed games must be ignored, we include the scores adjusted excluding the unrated games and the actual number N of played games.

For our convenience, we'll use the TPR values found in the previous exercise, including them in the crosstable for each opponent. Then we calculate the averages, excluding all the unplayed games.

#	NAME	TPR	SCORE	1	2	3	4	5	APRO
3	Charline	2089	3.5	=W11	+B6	+W8	=B4	=W2	1904
				1776	1813	1730	2081	2120	
4	David	2081	3.5	+B12	=BYE	+W13	=W3	=B1	1772
				1250	---	1781	2089	1969	
1	Alyx	1969	3.5	+W9	=B13	=W2	+B15	=W4	1789
				1175	1781	2120	1788	2081	
16	Stephan	1969	3.5	=W8	+B11	+W7	-B2	+W15	1805
				1730	1776	1611	2120	1788	

Sorting the crosstable based on APRO, the new ranking results (#3, #16, #1, #4), whereas ARO and TPR both yield (#3, #4, #1, 16) – the different choice of tie-breaks gives a different composition of the podium!

5.4 Perfect tournament performance (PTP)

The TPR provides an approximate indication of what the rating of a player should be to achieve the results they have actually attained. The idea behind PTP is to assess this value *as precisely as possible*, so that the expected score for that rating accurately corresponds to the achieved score; hence, the definition in [10.3]. This tie-break can be considered an improved version of the TPR and is a strength indicator that is certainly more reliable than those seen so far, even though it inevitably reflects the effects of any imprecise ratings. The drawback is that the process is quite laborious. It requires the repeated use of the table in B.02-8.1.2 and long and repeated sequences of calculations, making it suitable for computer calculation - but not at all for manual computation.

Exercise 22

In the Swiss tournament, determine the Perfect Performance of player #3.

For this calculation we need the results of the player and the opponents' ratings.

#	NAME	SCOR	ELO	ARO	TPR	1	2	3	4	5
3	Charline	3.5	2100	1940	2089	=W11	+B6	+W8	=B4	=W2
						1700	1950	1850	2050	2150

In general, the PTP value should not be too far from the TPR, which in this case is 2089. For want of better information, let's take this as a starting point. To begin with, let's calculate the score we'd expect if the player had this value as a rating, and compare it to the actual score (3.5) - in this calculation, *the ± 400 points cut does not apply*.

Opp.	Rating	Rating difference	dp	Pa
1	1700	2089 - 1700 =	+389	0.91
6	1950	2089 - 1950 =	+139	0.69
8	1850	2089 - 1850 =	+239	0.80
4	2050	2089 - 2050 =	+39	0.55
2	2150	2089 - 2150 =	-61	0.42
				3.37

The expected score we obtained is lower than actual score approximately by 4%, so we should increase our PTP estimate. For the next attempt, let's then increase it by about 4%, resulting in the value 2170, and repeat the calculation.

Opp.	Rating	Rating difference		dp	Pa
1	1700	2170 – 1700 =	+470	0.95	
6	1950	2170 – 1950 =	+220	0.78	
8	1850	2170 – 1850 =	+320	0.87	
4	2050	2170 – 2050 =	+120	0.66	
2	2150	2170 – 2150 =	+20	0.53	3.79

Now our guess is too high, we then lower it to 2130 – which is the intermediate value between the previous two guesses, and once again repeat the calculation.

Opp.	Rating	Rating difference		dp	Pa
1	1700	2130 – 1700 =	+430	0.93	
6	1950	2130 – 1950 =	+180	0.74	
8	1850	2130 – 1850 =	+280	0.84	
4	2050	2130 – 2050 =	+80	0.61	
2	2150	2130 – 2150 =	-20	0.47	3.59

With each iteration, the resulting expected score gets closer to the actual score, making the estimation of TPT more accurate. The process should continue until the smallest value that yields the desired result is obtained. To reach the final result, four more steps are required (omitted here), and the outcome is 2112.

Opp.	Rating	Rating difference		dp	Pa
1	1700	2112 – 1700 =	+412	0.93	
6	1950	2112 – 1950 =	+162	0.71	
8	1850	2112 – 1850 =	+262	0.82	
4	2050	2112 – 2050 =	+62	0.59	
2	2150	2112 – 2150 =	-38	0.45	3.50

As we anticipated, to obtain results in a reasonable time, the calculation of this tie-break *must* be entrusted to a computer. We should however mention that the method outlined here has been chosen for its simplicity but is not the only possible approach - and certainly not the most efficient.

It's worth noting that the calculation of the PTP is theoretically impossible for a player who has achieved a score of zero, because no rating difference results in a win probability of zero. Therefore, a figurative value must be assigned to this tie-break, which the regulations set at 800 points less than the lowest-rated opponent.

From this tie-break stems the Average Perfect Performance of Opponents (APPO) [10.5], which, similar to APRO for TPR, is the average of PTPs of opponents faced on-the-board. Calculating this tie-break requires precomputing the PTPs for many players and then taking the average, but we won't do that here.

To conclude the chapter, we present once again a summary table of rating-based tie-breaks. The final standings according to the various tie-break systems are highlighted with colour codes, and the table is sorted by score and ARO (e.g., brown corresponds to the second position, red to the third; if the ranking were ordered by TPT, the order of players #3 and #4 would be reversed; if ordered by APRO, players #4, #16, and #1 would be interchanged).

Here, it can be observed that the order of players #12, #15, and #14 is not the same between ARO and TPR, despite having the same score (see note on page 35). This is because, for the calculation of TPR (as well as for all other rating-based tie-breaks), unplayed games are ignored, regardless of the reason and the attributed score. For TPR calculations, the scores of these players are not equivalent (players #12 and #9 have zero points!), and consequently, their respective dp values are not equivalent either.

RNK	#	NAME	ELO	SCORE	ARO	TPR	APRO	PTP	APPO
1	2	Bruno	2150	4,0	1880	2120	1856	2216	1852
2	3	Charline	2100	3,5	1940	2089	1904	2112	1934
3	4	David	2050	3,5	1888	2081	1772	2168	1784
4	1	Alyx	2200	3,5	1820	1969	1789	2029	1769
5	16	Stephan	1450	3,5	1820	1969	1805	2013	1799
6	6	Franck	1950	3,0	1813	1813	1846	1810	1836
7	11	Maria	1700	2,5	1863	1776	1840	1763	1836
8	8	Irina	1850	2,5	1730	1730	1915	1715	1924
9	5	Helene	2000	2,5	1690	1690	1719	1689	1676
10	12	Nick (W)	1650	2,0	2050	1250	2081	1250	2168
11	15	Reine	1500	2,0	1860	1788	1776	1768	1767
12	14	Paul	1550	2,0	1800	1925	1775	1942	1756
13	9	Jessica	1800	1,5	1975	1175	1805	950	1802
14	13	Opal	1600	1,5	1930	1781	1879	1744	1909
15	7	Genevieve	1900	1,5	1760	1611	1869	1531	1890
16	10	Lais	1750	1,0	1880	1640	1717	1575	1687

Key:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
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6 TIE-BREAKS BASED ON DIRECT ENCOUNTER (DE)

Those tie-breaks compare the results the players attained in facing each other. First, we need to extract those games from the general crosstable – and only them – to create a separate ranking (sometimes called “head-to-head”). This ranking is then used to break ties. In Swiss tournaments, all unplayed or forfeited games are excluded- however, the tournament rules could include them in the calculation [6.1.1]. In round-robin or other tournaments with predetermined pairings, forfeits count as normal matches [15.2].

As with most tie-breaks, some players can still be tied, even after applying the separate ranking. The peculiarity of the direct encounter is that, in this case, the process is (repeatedly) applied to the players still tied, until they are all untied, or it is no longer possible to untie any. Another peculiarity is that this tie-break can be inserted multiple times in the list; for example, we could have the direct encounter, followed by the Buchholz, and then again direct encounter.

If all the tied players faced each other, their separate crosstable is just as if they had played a round-robin among themselves. In Swiss tournaments, this happens rather seldom, due to the fact that the crosstable often contains "gaps". Even in this case, a player may still be first, regardless of any possible outcome of the missing matches [6.3].

Let's consider a simple example (see the crosstable on the right). Alyx won against everyone, Bruno won against everyone but Alyx; Charline and David, on the other hand, did not play against each other – however, even if they had, they couldn't have reached Alyx's score. Alyx would still be the first in any case and is therefore ranked first. Once the first place is assigned, we observe that, regardless of the outcomes of the missing matches, no one could surpass Bruno either, so he is ranked second. However, certainty ends here – for the remaining players, we have no such assurances. At this point, the process restarts with the remaining players composing a new separate ranking until no more players can be untied (which is the case here, as only Charline and David are left, both with zero points). Then, we should proceed to the next tie-breaking system or drawing of lots.

	A	B	C	D
Alyx	*	1	1	1
Bruno	0	*	1	1
Charline	0	0	*	-
David	0	0	-	*

6.1 Direct encounter in Swiss tournaments

Exercise 23

In the Swiss tournament, rank all 3.5-points players using direct encounter (DE).

Let's extract the tied players' games from the general crosstable and compile the separate crosstable. David achieved the best results in the matches he played, but the crosstable includes several gaps. We need to investigate what could have happened, based on all the potential outcomes of the unplayed games. In theory, we should try all possible combinations of outcomes, but we don't really do all that. A useful practical method is to consider all the unplayed matches (in blue in the adjacent table) as wins. Even though this may yield some "impossible" scores, it makes readily apparent whether those matches could be decisive or not. Here, for example, it's clear that Stephan's score (and indeed anyone's) could have surpassed David's, so we don't have a definite winner, and no ties can be broken.

	A	C	D	S
Alyx	*	-	1/2	-
Charline	-	*	1/2	-
David	1/2	1/2	*	-
Stephan	-	-	-	*

	A	C	D	S
Alyx	*	1	1/2	1
Charline	1	*	1/2	1
David	1/2	1/2	*	1
Stephan	1	1	1	*

Exercise 24

In the Swiss tournament, rank all 3.5-points players using direct encounter (DE).

Once again, let's compile the separate crosstable. Now we have only one game, and once again no tie can be broken. incidentally, we observe that for the scoregroups at 2.0 and 1.5 points there are no played games at all, and therefore no tie can be broken there too.

	E	I	M
Helene	*	-	1
Irina	-	*	-
Maria	0	-	*

Exercise 25

Determine the standings of the Swiss tournament through the Direct Encounter tie-break.

For our convenience, let's highlight in different colours the four groups of tied players.

#	NAME	SCORE	1	2	3	4	5	6	7	8	9
3	Charline	6,5	=W11	+B6	+W8	=B4	-W2	+B16	=W1	+W14	+B10
1	Alyx	6,0	+W9	=B13	=W2	+B15	=W4	=B6	=B3	=W16	+B8
2	Bruno	6,0	+B10	+W7	=B1	+W16	+B3	=W4	-W6	=B15	=W5
4	David	6,0	+B12	=BYE	+W13	=W3	=B1	=B2	=W16	=B6	+W15
6	Franck	6,0	-B14	-W3	+BYE	+W10	+B8	=W1	+B2	=W4	+B16
7	Genevieve	4,5	+W15	-B2	-B16	=W5	-B11	-B10	+W13	+W9	+B14
8	Irina	4,5	=B16	+W14	-B3	+W13	-W6	=B11	=W15	+B5	-W1
16	Stephan	4,5	=W8	+B11	+W7	-B2	+W15	-W3	=B4	=B1	-W6
5	Helene	4,0	-W13	-B15	+W11	=B7	+W10	-B14	+B9	-W8	=B2
10	Lais	4,0	-W2	+B9	-W15	-B6	-B5	+W7	+B12	+W11	-W3
14	Paul	4,0	+W6	-B8	-F12	--	+B13	+W5	+W11	-B3	-W7
15	Reine	4,0	-B7	+W5	+B10	-W1	-B16	+W12	=B8	=W2	-B4
11	Maria	3,5	=B3	-W16	-B5	+F9	+W7	=W8	-B14	-B10	=W13
9	Jessica	3,0	-B1	-W10	=BYE	-F11	+BYE	+B13	-W5	-B7	=W12
12	Nick	3,0	-W4	+BYE	+F14	--	--	-B15	-W10	=B13	=B9
13	Opal	2,5	+B5	=W1	-B4	-B8	-W14	-W9	-B7	=W12	=B11

Let's begin with the 6.0-points scoregroup, which yields the separate crosstable reported on the side. Here we observe that Franck scored two points, and is first, Bruno scored one point and is last, whereas Alyx and David, who scored 1.5 points each, are still tied. We should then apply once again the procedure to those two players, building a new separate crosstable including only them.

(6 p.)

	A	B	D	F
Alyx	*	1/2	1/2	1/2
Bruno	1/2	*	1/2	0
David	1/2	1/2	*	1/2
Franck	1/2	1	1/2	*

This attempt does not bring any new information, as these players had a draw. Hence, we can't break this tie, which must be resolved with the subsequent tie-breaks.

	A	D
Alyx	*	1/2
David	1/2	*

In the second scoregroup, at 4.5 points, not all players met each other. From the results we observe that, had Irina played and won with Genevieve, she would have reached the same score as Stephan. We cannot determine a clear first place – and should then abandon this attempt and proceed to the next tie-break.

(4,5 p.)

	G	I	S
Genevieve	*	-	0
Irina	-	*	1/2
Stephan	1	1/2	*

In the third scoregroup, at 4.0 points, the situation is just the same. Finally, the last scoregroup, at 3.0 points, includes only two players who have drawn against each other.

(4 p.)

	E	L	P	R
Helene	*	1	1	0
Lais	0	*	-	0
Paul	0	-	*	-
Reine	1	1	-	*

All in all, once again, the tie-break based on direct encounters yields poor results. This is in fact rather expected; direct encounter tie-breaks have limited effectiveness. That's why its use is relatively infrequent, and often, when used, it appears as the first tie-break, followed by other methods with better differentiating power.

6.2 Direct encounter in round-robin tournaments

Exercise 26

Determine the standings of the round-robin tournament through Direct Encounter.

The application of this method to round-robin tournaments is not substantially different from the case of Swiss-system tournaments - it is however a bit simpler, because unplayed games are not a factor. Also, the composition of the separate crosstable is easier – in fact, we only need to remove from the general crosstable all the rows and columns related to players not involved in the tie-break.

Here, we have two scoregroups to be subject to tie-break, for each of which we need to create the separate crosstable.

In the first scoregroup (see crosstable to the right), Alyx is first, with two points. Bruno and Charline are still tied – we should therefore apply the method once again, to only those two players, but this is useless because they drew. Since we cannot break this tie, we need to proceed to the next tie-break.

	A	B	C
Alyx	*	1	1
Bruno	0	*	1/2
Charline	0	1/2	*

For the second scoregroup, we need to remember that the forfeit win is considered just as any regularly played game [15.2]. In this case we can compose a complete ranking, with Helene first with 1.5 points, followed by Franck with 1 point and David with 1/2 point.

	D	E	F
David	*	1/2	0
Helene	1/2	*	+
Franck	1	-	*

7 OTHER TIE-BREAK SYSTEMS FOR INDIVIDUAL TOURNAMENTS

In this chapter, we will discuss individual tie-breaks of the "B" type ([7]), which only use the player's results. Those tie-breaks allow players to calculate or predict their tie-break value while still playing the round (which is impossible, for example, with the Buchholz, which depends on the results the previous opponents will obtain). These tie-breaks are all very easy to calculate and mostly ignore unplayed games. Some of them are very similar to each other, so we will discuss them together.

7.1 Number of wins (WIN) and Number of games won (WON)

The only difference between these two tie-breaks is that the first one (WIN) considers all games for which a score equal to that assigned for a win has been given, including PAB (Pairing Allocated Bye) and forfeit (and also any full-point byes, which are deprecated by the regulations but could still be awarded in certain specific cases). The second one (WON), on the other hand, only considers the games actually won on-the-board.

Exercise 27

Determine the standings of the Swiss tournament through the WIN tie-break system.

For this tie-break we count the number of games which ended with *a score corresponding to a win* – independent of the nature of the win. The tie-break value is the total count.

#	NAME	ELO	SCORE	1	2	3	4	5	WIN
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	3
				1	1	0	1	0	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	3
				0	1	1	0	1	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	2
				1	0	0	1	0	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	2
				0	1	1	0	0	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	2
				1	0	1	0	0	
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	3
				0	0	1	1	1	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	2
				0	0	1	0	1	
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	2
				0	1	0	1	0	
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	2
				0	0	0	1	1	
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	2
				0	1	1	0	0	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	2
				1	0	0	0	1	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	2
				0	1	1	0	0	
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	1
				1	0	0	0	0	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	1
				0	0	0	0	1	
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	1
				1	0	0	0	0	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	1
				0	1	0	0	0	

The practical application of the method is straightforward, we only need to pay attention to unplayed games. For example, among the results obtained by player #9, only the last of their three unplayed games counts, because this is a PAB and therefore is equivalent to a win. On the contrary, the other two unplayed games – a half point bye (HPB) and a forfeit loss – are not wins and therefore are not counted.

Exercise 28

Determine the standings of the Swiss tournament through the WON tie-break system.

For this tie-break we count the number of games *actually won on-the-board*. The tie-break value is the total count.

#	NAME	ELO	SCORE	1	2	3	4	5	WON
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	3
				1	1	0	1	0	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	3
				0	1	1	0	1	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	2
				1	0	0	1	0	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	2
				0	1	1	0	0	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	2
				1	0	1	0	0	
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	2
				0	0	0	1	1	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	2
				0	0	1	0	1	
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	2
				0	1	0	1	0	
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	1
				0	0	0	0	1	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	2
				1	0	0	0	1	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	2
				0	1	1	0	0	
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	0
				0	0	0	0	0	
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	1
				1	0	0	0	0	
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	1
				1	0	0	0	0	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	0
				0	0	0	0	0	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	1
				0	1	0	0	0	

Again, the practical application of the method is straightforward. Contrary to WIN system, all unplayed games are ignored (e.g., see player #9).

7.2 Number of games played with Black (BPG) and won with Black (BWG)

The first of these tie-breaks (BPG) considers only the games actually played with Black, while *all unplayed games are ignored*. The second one (BWG) only counts the won games. The underlying principle is that playing with Black is more difficult than playing with White and does therefore deserve to be rewarded.

Exercise 29

Determine the standings of the Swiss tournament through the BPG tie-break system.

For this tie-break we count the number of games *actually played* on-the-board with Black. The tie-break value is the total count.

#	NAME	ELO	SCORE	1	2	3	4	5	BPG
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	3
				1	0	1	0	1	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	2
				0	1	0	1	0	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	2
				0	1	0	1	0	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	2
				1	0	0	0	1	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	2
				0	1	0	1	0	
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	2
				1	0	0	0	1	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	2
				0	1	0	1	0	
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	2
				1	0	1	0	0	
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	2
				1	0	1	0	0	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	3
				1	0	1	0	1	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	2
				0	1	0	0	1	
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	0
				0	0	0	0	0	
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	3
				0	1	1	0	1	
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	3
				1	0	1	1	0	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	1
				1	0	0	0	0	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	3
				0	1	0	1	1	

Exercise 30

Determine the standings of the Swiss tournament through the BWG tie-break system.

For this tie-break we count the number of games *actually won* on-the-board with Black. The tie-break value is the total count.

#	NAME	ELO	SCORE	1	2	3	4	5	BWG
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	1
				1	0	0	0	0	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	1
				0	0	0	1	0	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	1
				0	1	0	0	0	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	1
				1	0	0	0	0	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	1
				0	1	0	0	0	

6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	1
				0	0	0	0	1	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	0
				0	0	0	0	0	
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	0
				0	0	0	0	0	
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	0
				0	0	0	0	0	
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	0
				0	0	0	0	0	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	1
				0	0	0	0	1	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	1
				0	0	1	0	0	
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	0
				0	0	0	0	0	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	0
				0	0	0	0	0	
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	1
				1	0	0	0	0	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	1
				0	1	0	0	0	

7.3 Games one elected to play (GE)

The logic of this tie-break is to penalize players who have chosen to play fewer games than others; the counting is trivial, we just need to be careful about which games not to count (a requested bye or a forfeit loss doesn't count, but a forfeited win does).

Exercise 31

Determine the standings of the Swiss tournament through the GE tie-break system.

In this tie-break we count the number of rounds in which the player was available to play, including possible games that were not played for reasons beyond the player's control. The tie-break value is the total count.

#	NAME	ELO	SCORE	1	2	3	4	5	GE
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	5
				1	1	1	1	1	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	5
				1	1	1	1	1	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	5
				1	1	1	1	1	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	5
				1	1	1	1	1	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	4
				1	0	1	1	1	
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	5
				1	1	1	1	1	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	5
				1	1	1	1	1	

8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	5
				1	1	1	1	1	
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	5
				1	1	1	1	1	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	5
				1	1	1	1	1	
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	3
				1	1	1	0	0	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	3
				1	1	0	0	1	
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	5
				1	1	1	1	1	
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	5
				1	1	1	1	1	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	3
				1	1	0	0	1	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	5
				1	1	1	1	1	

We want to note the different handling between "voluntary absences" (VUR) and the games in which the player is available (unplayed games are highlighted in red and blue).

7.4 Sum of progressive scores (PS)

This tie-break is the sum of the player's scores at the end of each round, regardless of whether they played or not. The score from each round is added as many times as there are rounds remaining in the tournament, so the more advanced the round is, the less the result of a round weighs on the total.

Modifiers like Cut-1 or Cut-2 can be applied to this tie-break. Since the least significant addend is always the one from the first round (this is the minimum possible score), applying Cut-1 involves subtracting the result of the first round (which, however, continues to contribute as part of the scores in the subsequent rounds).

Exercise 32

Determine the standings of the Swiss tournament through the PS tie-break system.

We calculate the player's score after each round, then sum all the scores.

#	NAME	ELO	SCORE	1	2	3	4	5	PS
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	13.0
				1.0	2.0	2.5	3.5	4.0	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	11.5
				1.0	1.5	2.5	3.0	3.5	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	11.0
				1.0	1.5	2.0	3.0	3.5	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	11.0
				0.5	1.5	2.5	3.0	3.5	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	10.5
				0.5	1.5	2.5	2.5	3.5	
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	6.0
				0.0	0.0	1.0	2.0	3.0	

8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	8.5
				0.5	1.5	1.5	2.5	2.5	
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	5.5
				0.5	0.5	0.5	1.5	2.5	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	5.0
				0.0	0.0	1.0	1.5	2.5	
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	7.0
				0.0	1.0	2.0	2.0	2.0	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	7.0
				0.0	1.0	2.0	2.0	2.0	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	6.0
				1.0	1.0	1.0	1.0	2.0	
13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	7.0
				1.0	1.5	1.5	1.5	1.5	
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	6.0
				1.0	1.0	1.0	1.5	1.5	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	2.5
				0.0	0.0	0.5	0.5	1.5	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	4.0
				0.0	1.0	1.0	1.0	1.0	

Exercise 33

Determine the standings of the Swiss tournament through the PS-C1 tie-break system.

In this tie-break we need to calculate the score at the end of each round, then add all those scores but the smallest – which is of course the one after the first round.

#	NAME	ELO	SCORE	1	2	3	4	5	PS-C1
2	Bruno	2150	4.0	+B10	+W7	=B1	+W16	=B3	12.0
				1.0	2.0	2.5	3.5	4.0	
3	Charline	2100	3.5	=W11	+B6	+W8	=B4	=W2	10.5
				0.5	1.5	2.5	3.0	3.5	
4	David	2050	3.5	+B12	=BYE	+W13	=W3	=B1	10.5
				1.0	1.5	2.5	3.0	3.5	
1	Alyx	2200	3.5	+W9	=B13	=W2	+B15	=W4	10.0
				1.0	1.5	2.0	3.0	3.5	
16	Stephan	1450	3.5	=W8	+B11	+W7	-B2	+W15	10.0
				0.5	1.5	2.5	2.5	3.5	
6	Franck	1950	3.0	-B14	-W3	+BYE	+W10	+B8	6.0
				0.0	0.0	1.0	2.0	3.0	
8	Irina	1850	2.5	=B16	+W14	-B3	+W13	-W6	8.0
				0.5	1.5	1.5	2.5	2.5	
5	Helene	2000	2.5	-W13	-B15	+W11	=B7	+W10	5.0
				0.0	0.0	1.0	1.5	2.5	
11	Maria	1700	2.5	=B3	-W16	-B5	+F9	+W7	5.0
				0.5	0.5	0.5	1.5	2.5	
12	Nick (W)	1650	2.0	-W4	+BYE	+F14	--	--	7.0
				0.0	1.0	2.0	2.0	2.0	
15	Reine	1500	2.0	-B7	+W5	+B10	-W1	-B16	7.0
				0.0	1.0	2.0	2.0	2.0	
14	Paul	1550	2.0	+W6	-B8	-F12	--	+B13	5.0
				1.0	1.0	1.0	1.0	2.0	

13	Opal	1600	1.5	+B5	=W1	-B4	-B8	-W14	6.0
				1.0	1.5	1.5	1.5	1.5	
7	Genevieve	1900	1.5	+W15	-B2	-B16	=W5	-B11	5.0
				1.0	1.0	1.0	1.5	1.5	
9	Jessica	1800	1.5	-B1	-W10	=BYE	-F11	+BYE	2.5
				0.0	0.0	0.5	0.5	1.5	
10	Lais	1750	1.0	-W2	+B9	-W15	-B6	-B5	4.0
				0.0	1.0	1.0	1.0	1.0	

Let's conclude this chapter comparing the results of the different systems. The following table shows the values resulting from each tie-break. The background colour of the cell represents the player's placement in the final ranking obtained with that system (for example, a black background in a cell indicates that, with that tie-break, the player would finish in the first position, and so on). Note that, among these tie-breaks, only those based on progressive scores (PS, PS-Cut 1) have a certain discriminatory power, while all other tie-breaks result in several residual ties.

RNK	#	NAME	ELO	SCORE	WIN	WON	BPG	BWG	GE	PS	PS-C1
1	2	Bruno	2150	4,0	3	3	3	1	5	13,0	12,0
2	4	David	2050	3,5	2	2	2	1	4	11,5	10,5
3	1	Alyx	2200	3,5	2	2	2	1	5	11,0	10,0
4	3	Charline	2100	3,5	2	2	2	1	5	11,0	10,5
5	16	Stephan	1450	3,5	3	3	2	1	5	10,5	10,0
6	6	Franck	1950	3,0	3	2	2	1	5	6,0	6,0
7	8	Irina	1850	2,5	2	2	2	0	5	8,5	8,0
8	11	Maria	1700	2,5	2	1	2	0	5	5,5	5,0
9	5	Helene	2000	2,5	2	2	2	0	5	5,0	5,0
10	12	Nick (W)	1650	2,0	2	0	0	0	3	7,0	7,0
11	15	Reine	1500	2,0	2	2	3	1	5	7,0	7,0
12	14	Paul	1550	2,0	2	2	2	1	3	6,0	5,0
13	13	Opal	1600	1,5	1	1	3	1	5	7,0	6,0
14	7	Genevieve	1900	1,5	1	1	3	0	5	6,0	5,0
15	9	Jessica	1800	1,5	1	0	1	0	3	2,5	2,5
16	10	Lais	1750	1,0	1	1	3	1	5	4,0	4,0

Key: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

PART TWO – TEAM TOURNAMENTS

To compile the individual rankings of players in team tournaments, all the tie-break systems mentioned can be used. Additionally, other methods are needed to determine the order of placement for each team, introducing some more complexity compared to individual tournaments. The most obvious difference is the existence of two parallel scoring systems, team points (MP) and game points (GP), and many tie-breaks can be calculated by choosing either one of the two – and, in certain cases, even combining them (such as in Sonneborn-Berger). The score used for pairing, known as the *primary score*, is often (but not necessarily) also used as the first criterion for ranking. The other score, known as the *secondary score*, can be used in tie-breaks.

The tournament rules must specify which of the two is the primary score and which is secondary, and whether and how the latter should be used in pairings.

The calculation of tie-breaks requires extensive information. The following data are required (see paragraph 2.3, page 6 and following):

- Team composition
- Team (match) pairings
- Individual games pairings (this can be quite extensive)
- Sometimes, the board on which each game was played (board order)

While the first three are usually provided directly by the pairing software, the fourth usually is not immediately available and needs to be inferred from the pairings. In some tournaments, player placement follows the rating order, simplifying the process. In tournaments where board order is free, obtaining this information can be rather tedious.

Forfeits are rarer in team tournaments than in individual ones (but nonetheless they exist). There is no substantial difference in managing unplayed games or matches compared to individual tournaments.

In calculating tie-breaks for team tournaments, even more than in individual, it is recommended to use a spreadsheet if possible. This calculation may be required when the tournament management program does not handle the required tie-break, or a verification is necessary.

The scoring system used for team points in all examples is the traditional 2-1-0.

8 MATCH POINTS VERSUS GAME POINTS (MPvGP)

This tie-break uses the secondary score to break ties left by the primary score. According to the tournament rules, team points (MP) can be used to break ties in the individual points (GP) standings, or (more often) vice versa.

Exercise 34

Compile tournament standings using a) match points or b) game points as the primary score.

The tournament management software usually provides those scores (more or less) automatically. Therefore, the application of this tie-break system is straightforward and only requires reordering the standings. The first table displays teams ordered by {MP, GP}, while the second shows the ordering {GP, MP}.

#	TEAM	MP	GP	R1	R2	R3	R4	R5	R6	R7
5	Elephants	10	18	12w4	6b3	2w4	4b½	1b2½	3w½	13b3½
1	Antelopes	10	17,5	8w2½	4b1½	7w3	2b2½	5w1½	+F	3b2½
2	Bonobos	10	17	9b3	3w4	5b0	1w1½	13w3	4b2½	10b3
4	Deer	10	17	11b3	1w2½	13b3	5w3½	3b1	2w1½	9b2½
3	Cougars	10	16	10w2½	2b0	9w3	6b2½	4w3	5b3½	1w1½
8	Hippopotami	7	15	1b1½	7w2	6w1	12b3½	10b2	9w2	11w3
6	Falcons	7	12,5	13b2	5w1	8b3	3w1½	11b2½	-F	12w2½
9	Iguanas	6	14,5	2w1	12b4	3b1	14w3	7w2	8b2	4w1½
7	Giraffes	6	11,5	14w2	8b2	1b1	10w2½	9b2	13w2	ZPB
13	Moose	6	11,5	6w2	14b2½	4w1	11w2½	2b1	7b2	5w½
10	Jackals	5	13	3b1½	11w1½	12b2½	7b1½	8w2	14b3	2w1
11	Koalas	4	11,5	4w1	10b2½	14w2	13b1½	6w1½	12w2	8b1
14	Narwhals	4	11,5	7b2	13w1½	11b2	9b1	HPB	10w1	PAB
12	Lynxes	2	7,5	5b0	9w0	10w1½	8w½	PAB	11b2	6b1½

#	TEAM	GP	MP	R1	R2	R3	R4	R5	R6	R7
5	Elephants	18	10	12w4	6b3	2w4	4b½	1b2½	3w½	13b3½
1	Antelopes	17,5	10	8w2½	4b1½	7w3	2b2½	5w1½	+F	3b2½
2	Bonobos	17	10	9b3	3w4	5b0	1w1½	13w3	4b2½	10b3
4	Deer	17	10	11b3	1w2½	13b3	5w3½	3b1	2w1½	9b2½
3	Cougars	16	10	10w2½	2b0	9w3	6b2½	4w3	5b3½	1w1½
8	Hippopotami	15	7	1b1½	7w2	6w1	12b3½	10b2	9w2	11w3
9	Iguanas	14,5	6	2w1	12b4	3b1	14w3	7w2	8b2	4w1½
10	Jackals	13	5	3b1½	11w1½	12b2½	7b1½	8w2	14b3	2w1
6	Falcons	12,5	7	13b2	5w1	8b3	3w1½	11b2½	-F	12w2½
7	Giraffes	11,5	6	14w2	8b2	1b1	10w2½	9b2	13w2	ZPB
13	Moose	11,5	6	6w2	14b2½	4w1	11w2½	2b1	7b2	5w½
11	Koalas	11,5	4	4w1	10b2½	14w2	13b1½	6w1½	12w2	8b1
14	Narwhals	11,5	4	7b2	13w1½	11b2	9b1	HPB	10w1	PAB
12	Lynxes	7,5	2	5b0	9w0	10w1½	8w½	PAB	11b2	6b1½

The resulting standings are in general rather different and could reward (or punish) different teams. This is why the choice of the primary score system must *always* be made explicit before the tournament starts.

9 SISTEMA BUCHHOLZ (BH)

Once it's determined which score to use (MP or GP), there is no substantial difference between calculating the Buchholz for a team and that for an individual player in an individual tournament.

Exercise 35

Using match points (MP) as primary score, compile the final standings of the tournament using Buchholz system (total).

For each participant and round, let's enter the scores of all opposing teams (MP or GP depending on the case – here both are shown, just as an example) into the team pairings crosstable (see table below).

#	Team	MP	GP	R1	R2	R3	R4	R5	R6	R7
1	Antelopes	10	17½	8w2½	4b1½	7w3	2b2½	5w1½	+F	3b2½
	Opponent MP			7	10	6	10	10	10	10
	Opponent GP			15,0	17,0	11,5	17,0	18,0	17,5	16,0
2	Bonobos	10	17	9b3	3w4	5b0	1w1½	13w3	4b2½	10b3
	Opponent MP			6	10	10	10	6	10	5
	Opponent GP			14,5	16,0	18,0	17,5	11,5	17,0	13,0

Team #1 won a match by forfeit – the outcome of this match is therefore calculated as a win against a dummy opponent with the same score and result, just like in the individual case [16.4] (except, of course, that the dummy opponent is now a team, not a player). Therefore, in the corresponding cell, we enter the score (MP or GP) of the team itself.

As usual, we need to be careful with matches "unplayed on request" that are not followed by any round with availability to play [16.2.5]. In calculating the tie-break of the opposing teams, these matches should be considered as draws [16.3.2]. In our tournament, this happens in the last round for team #7, whose score needs to be adjusted for all the opponents it faced (but not for the team itself).

#	Team	MP	GP	R1	R2	R3	R4	R5	R6	R7	TOT
7	Giraffes	6	11½	14w2	8b2	1b1	10w2½	9b2	13w2	ZPB	
	Adjusted MP			1	1	0	2	1	1	1	7
	Adjusted GP			2	2	1	2,5	2	2	2	13,5

We can then compile a table in which, for each team, we list the score (here, MP) of the opposing team encountered in that round, adjusted as described above. This is the only score to adjust in our crosstable – all other unplayed matches are calculated at face value.

We thus obtain the following table, where the last column shows the sum of contributes per round, which is the final value of the Buchholz (matches for which the adjusted score of the opposing team is used are highlighted, and the table is already sorted by decreasing score and Buchholz). Note that, for the last round of team #7, which is a requested bye, we use the actual rather than the adjusted score [16.4].

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	BH
1	Antelopes	10	17,5	10	17,5	7	10	7	10	10	10	10	64
3	Cougars	10	16,0	10	16,0	5	10	6	7	10	10	10	58
2	Bonobos	10	17,0	10	17,0	6	10	10	10	6	10	5	57
4	Deer	10	17,0	10	17,0	4	10	6	10	10	10	6	56
5	Elephants	10	18,0	10	18,0	2	7	10	10	10	10	6	55
6	Falcons	7	12,5	7	12,5	6	10	7	10	4	7	2	46
8	Hippopotami	7	15,0	7	15,0	10	7	7	2	5	6	4	41

13	Moose	6	11,5	6	11,5	7	4	10	4	10	7	10	52
9	Iguanas	6	14,5	6	14,5	10	2	10	4	7	7	10	50
7	Giraffes	6	11,5	7	13,5	4	7	10	5	6	6	6	44
10	Jackals	5	13,0	5	13,0	10	4	2	7	7	4	10	44
11	Koalas	4	11,5	4	11,5	10	5	4	6	7	2	7	41
14	Narwhals	4	11,5	4	11,5	7	6	4	6	4	5	4	36
12	Lynxes	2	7,5	2	7,5	10	6	5	7	2	4	7	41

(Note: the fact that all ties here are resolved is but a *lucky coincidence*.)

Exercise 36

Using match points (MP) as primary score, compile the final standings of the tournament using Buchholz system Cut-1 (BH-C1).

The calculation is very similar to the previous one, we just need to discard the contribute owed to the *least significant value* in each sum. We want to take care of possible forfeit losses (#6, sixth round) or requested byes (#14, fifth round, and #7, seventh round), because the contributes from these matches are the first to be cut. The cut contributes are highlighted in colour (light red for less significant values, yellow for those due to voluntary absences as mentioned above).

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	BH-C1
1	Antelopes	10	17,5	10	17,5	7	10	7	10	10	10	10	57
3	Cougars	10	16,0	10	16,0	5	10	6	7	10	10	10	53
5	Elephants	10	18,0	10	18,0	2	7	10	10	10	10	6	53
2	Bonobos	10	17,0	10	17,0	6	10	10	10	6	10	5	52
4	Deer	10	17,0	10	17,0	4	10	6	10	10	10	6	52
6	Falcons	7	12,5	7	12,5	6	10	7	10	4	7	2	39
8	Hippopotami	7	15,0	7	15,0	10	7	7	2	5	6	4	39
9	Iguanas	6	14,5	6	14,5	10	2	10	4	7	7	10	48
13	Moose	6	11,5	6	11,5	7	4	10	4	10	7	10	48
7	Giraffes	6	11,5	7	13,5	4	7	10	5	6	6	6	38
10	Jackals	5	13,0	5	13,0	10	4	2	7	7	4	10	42
11	Koalas	4	11,5	4	11,5	10	5	4	6	7	2	7	39
14	Narwhals	4	11,5	4	11,5	7	6	4	6	4	5	4	32
12	Lynxes	2	7,5	2	7,5	10	6	5	7	2	4	7	39

We should observe that the cut value due to voluntary absences is often greater than the least significant value, and that is correct. *Rule [16.5]* is designed to prevent a competitor from gaining an advantage by choosing a voluntary absence rather than playing the match.

Exercise 37

Using game points (GP) as primary score, compile the final standings of the tournament using Buchholz system Cut-1.

The only practical difference from the previous examples is the use of a different score. So, this time we start from the table reordered for decreasing GP and enter the GP scores (adjusted) of the opposing teams for each round.

#	Team	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	BH GP	BH-C1
1	Antelopes	10	17,5	10	17,5	15,0	17,0	13,5	17,0	18,0	17,5	16,0	114,0	100,5
3	Cougars	10	16,0	10	16,0	13,0	17,0	14,5	12,5	17,0	18,0	17,5	109,5	97,0
2	Bonobos	10	17,0	10	17,0	14,5	16,0	18,0	17,5	11,5	17,0	13,0	107,5	96,0
4	Deer	10	17,0	10	17,0	11,5	17,5	11,5	18,0	16,0	17,0	14,5	106,0	94,5
5	Elephants	10	18,0	10	18,0	7,5	12,5	17,0	17,0	17,5	16,0	11,5	99,0	91,5
8	Hippopotami	7	15,0	7	15,0	17,5	13,5	12,5	7,5	13,0	14,5	11,5	90,0	82,5
6	Falcons	7	12,5	7	12,5	11,5	18,0	15,0	16,0	11,5	12,5	7,5	92,0	79,5
9	Iguanas	6	14,5	6	14,5	17,0	7,5	16,0	11,5	13,5	15,0	17,0	97,5	90,0
13	Moose	6	11,5	6	11,5	12,5	11,5	17,0	11,5	17,0	13,5	18,0	101,0	89,5
7	Giraffes	6	11,5	7	13,5	11,5	15,0	17,5	13,0	14,5	11,5	11,5	94,5	83,0
10	Jackals	5	13,0	5	13,0	16,0	11,5	7,5	13,5	15,0	11,5	17,0	92,0	84,5
11	Koalas	4	11,5	4	11,5	17,0	13,0	11,5	11,5	12,5	7,5	15,0	88,0	80,5
14	Narwhals	4	11,5	4	11,5	13,5	11,5	11,5	14,5	11,5	13,0	11,5	87,0	75,5
12	Lynxes	2	7,5	2	7,5	18,0	14,5	13,0	15,0	7,5	11,5	12,5	92,0	84,5

10 EXTENDED SONNEBORN-BERGER SYSTEM FOR TEAMS (ESB)

The extended Sonneborn-Berger system for teams (ESB) is calculated by summing, for each team, the product of the total score of each opponent (at the end of the tournament) by the score obtained by the team itself against that opponent. Since there are two different scoring systems (MP or GP), there are four possible combinations, depending on the choice of the score to use for the competing team and the opponents. The table below summarizes the possibilities.

		Opponent score	
		MP	GP
Own score	MP	EMMSB Opponent MP × MP obtained	EGMSB Opponent GP × MP obtained
	GP	EMGSB Opponent MP × GP obtained	EGGSB Opponent GP × GP obtained

We can use any of these, or any combination. The ESB can be subject to "Cut" modifiers (notably, Cut-1). On the contrary, "Median" type modifiers do not apply because this tie-break, by its nature, aims to give more importance to results scored against stronger opponents, so it wouldn't make sense to ignore just them.

The ESB tie-break can be used with either round-robin or Swiss-system tournaments. In the latter case, unplayed games and matches, just as in the case of SB for individual tournaments, are given different values based on the type of absence [16].

We will see presently several examples of application in the Swiss tournament; everything shown can be directly extended to round-robin, except for the handling of unplayed matches or games, which in the latter case are treated just as if they had been played.

Exercise 38

Primary score is MP. Calculate EMMSB-Cut 1 for all teams at 10 points.

Let's extract from the general crosstable the data relevant to the five tied teams.

#	TEAM	MP	GP	R1	R2	R3	R4	R5	R6	R7
1	Antelopes	10	17½	8w2½	4b1½	7w3	2b2½	5w1½	+F	3b2½
2	Bonobos	10	17	9b3	3w4	5b0	1w1½	13w3	4b2½	10b3
3	Cougars	10	16	10w2½	2b0	9w3	6b2½	4w3	5b3½	1w1½
4	Deer	10	17	11b3	1w2½	13b3	5w3½	3b1	2w1½	9b2½
5	Elephants	10	18	12w4	6b3	2w4	4b½	1b2½	3w½	13b3½
6	Falcons	7	12½	13b2	5w1	8b3	3w1½	11b2½	-F	12w2½
7	Giraffes	6	11½	14w2	8b2	1b1	10w2½	9b2	13w2	ZPB
8	Hippopotami	7	15	1b1½	7w2	6w1	12b3½	10b2	9w2	11w3
9	Iguanas	6	14½	2w1	12b4	3b1	14w3	7w2	8b2	4w1½
10	Jackals	5	13	3b1½	11w1½	12b2½	7b1½	8w2	14b3	2w1
11	Koalas	4	11½	4w1	10b2½	14w2	13b1½	6w1½	12w2	8b1
12	Lynxes	2	7½	5b0	9w0	10w1½	8w½	PAB	11b2	6b1½
13	Moose	6	11½	6w2	14b2½	4w1	11w2½	2b1	7b2	5w½
14	Narwhals	4	11½	7b2	13w1½	11b2	9b1	HPB	10w1	PAB

For the calculation, we need the MP scores of the teams involved and their opponents, and it is recommended to first gather all the necessary data. In the table below, the first row shows the "team's card", inferred from the crosstable. The second row lists the MP scores obtained by the team against each opponent, already adjusted for any unplayed matches (as seen in previous exercises, this adjustment concerns only the matches with team #7, because all other voluntary absences are followed by rounds with availability to play, see [16.3]). The third row, on the other hand, shows the MP (total) scores of opponents faced (real or dummy). Finally, the fourth row shows the products between the score obtained against the opposing team and the score of the latter – these are the contributions that, when summed together, and excluding the one corresponding to the least significant value, finally give the value of the tie-break. Here, the contribution to be discarded, which is not the smallest but that related to the least significant opponent (i.e., the one with the lower score), is highlighted (light red background). Finally, the last column shows the values of the total tie-break (above) and the Cut-1 tie-break, obtained by discarding the least significant value (opponent). All of this is, of course, repeated for each team to untie.

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMMSB Cut-1
1	Antelopes	10	17½	10	17½	8w2½	4b1½	7w3	2b2½	5w1½	+F	3b2½	88 74
	<i>Team MP</i>					2	0	2	2	0	2		
	<i>Opponent MP</i>					7	10	7	10	10	10		
	<i>ESB Contribute</i>					14	0	14	20	0	20		

Firstly, let's note the adjusted score attributed to the opponent #7 (R3), which asked for a ZPB in the last round. Also, observe that team #1 has an unplayed match in the sixth round, to be counted at face value [16.4].

Team #1 had two opponents (#7 and #8) with the same minimum score. We should exclude the one against which the team obtained the worst result, but in this case, those are equal too. Therefore, we can choose either one of the two. The total value of the EMMSB tie-break is determined by:

$$\text{EMMSB} = 2 \times 7 + 0 \times 10 + 2 \times 7 + 2 \times 10 + 0 \times 10 + 2 \times 10 + 2 \times 10 = 14 + 0 + 14 + 20 + 0 + 20 + 20 = 88$$

Discarding the contribute corresponding to the least significant value (i.e., that of team #7 or #8), we have:

$$\text{EMMSB-C1} = 0 \times 10 + 2 \times 7 + 2 \times 10 + 0 \times 10 + 2 \times 10 + 2 \times 10 = 14 + 0 + 14 + 20 + 0 + 20 + 20 = 74$$

Once again, let's focus our attention on this point – as clearly seen in this case, the contribute related to the least significant value in general is not the smaller one.

Results for other teams are calculated in the very same way. The calculations are left as an exercise for the reader.

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMMSB Cut-1
2	Bonobos	10	17	10	17	9b3	3w4	5b0	1w1½	13w3	4b2½	10b3	74 64
	<i>Team MP</i>					2	2	0	0	2	2	2	
	<i>Opponent MP</i>					6	10	10	10	6	10	5	
	<i>ESB Contribute</i>					12	20	0	0	12	20	10	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMMSB Cut-1
3	Cougars	10	16	10	16	10w2½	2b0	9w3	6b2½	4w3	5b3½	1w1½	76 66
	<i>Team MP</i>					2	0	2	2	2	2	0	
	<i>Opponent MP</i>					5	10	6	7	10	10	10	
	<i>ESB Contribute</i>					10	0	12	14	20	20	0	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMMSB Cut-1
4	Deer	10	17	10	17	11b3	1w2½	13b3	5w3½	3b1	2w1½	9b2½	72 64
	<i>Team MP</i>					2	2	2	2	0	0	2	
	<i>Opponent MP</i>					4	10	6	10	10	10	6	
	<i>ESB Contribute</i>					8	20	12	20	0	0	12	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMMSB Cut-1
5	Elephants	10	18	10	18	12w4	6b3	2w4	4b½	1b2½	3w½	13b3½	70 66
	<i>Team MP</i>					2	2	2	0	2	0	2	
	<i>Opponent MP</i>					2	7	10	10	10	10	6	
	<i>ESB Contribute</i>					4	14	20	0	20	0	12	

In the end, teams #3 and #5, still tied, share the second and third place, while teams #2 and #4 share the fourth and fifth place (incidentally, the SB total tie-break would not have left unresolved ties, but this is just a lucky coincidence).

Exercise 39

Primary score is MP. Calculate EGMSB for all teams at 10 points.

Let's extract from the general crosstable the data relevant to the tied teams (see previous examples). For this calculation, we need MP scores of the involved teams and the GP scores of their opponents. Numbers are now different (GP score is typically greater than MP) but the calculation method is identical to that seen in the previous exercise.

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EGMSB
1	Antelopes	10	17½	10	17½	8w2½	4b1½	7w3	2b2½	5w1½	+F	3b2½	158,0
	Team MP					2	0	2	2	0	2		
	Opponent GP					15,0	17,0	13,5	17,0	18,0	17,5	16,0	
	ESB Contribute					30,0	0,0	27,0	34,0	0,0	35,0	32,0	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EGMSB
2	Bonobos	10	17	10	17	9b3	3w4	5b0	1w1½	13w3	4b2½	10b3	144,0
	Team MP					2	2	0	0	2	2	2	
	Opponent GP					14,5	16,0	18,0	17,5	11,5	17,0	13,0	
	ESB Contribute					29,0	32,0	0,0	0,0	23,0	34,0	26,0	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EGMSB
3	Cougars	10	16	10	16	10w2½	2b0	9w3	6b2½	4w3	5b3½	1w1½	150,0
	Team MP					2	0	2	2	2	2	0	
	Opponent GP					13,0	17,0	14,5	12,5	17,0	18,0	17,5	
	ESB Contribute					26,0	0,0	29,0	25,0	34,0	36,0	0,0	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EGMSB
4	Deer	10	17	10	17	11b3	1w2½	13b3	5w3½	3b1	2w1½	9b2½	146,0
	Team MP					2	2	2	2	0	0	2	
	Opponent GP					11,5	17,5	11,5	18,0	16,0	17,0	14,5	
	ESB Contribute					23,0	35,0	23,0	36,0	0,0	0,0	29,0	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EGMSB
5	Elephants	10	18	10	18	12w4	6b3	2w4	4b½	1b2½	3w½	13b3½	132,0
	Team MP					2	2	2	0	2	0	2	
	Opponent GP					7,5	12,5	17,0	17,0	17,5	16,0	11,5	
	ESB Contribute					15,0	25,0	34,0	0,0	35,0	0,0	23,0	

Now there are no more tied teams – this makes sense because the values of GP scores are more varied, and therefore different results have greater probability. We also want to observe that the least significant contributions are not the same as in the previous case.

Exercise 40

Primary score is MP. Calculate EMGSB for all teams at 6 points.

Let's extract from the general crosstable the data relevant to the tied teams (see previous examples). For this calculation, we need GP scores of the involved teams and the MP scores of their opponents. Once again, the calculation method is identical to that seen in the previous exercise.

The case of team #7 highlights that, in the Sonneborn-Berger system, there is no need for special considerations for forfeit losses and zero-point byes (pre-announced absences), as *their contribution is always null*. However, it remains necessary to consider the half-point byes, which fall under the category of voluntary absences but provide a non-zero contribution. (These may need to be discarded when applying the Cut-1 modifier.)

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMGSB
7	Giraffes	6	11½	7	13½	14w2	8b2	1b1	10w2½	9b2	13w2	ZPB	68,5
	Team GP					2,0	2,0	1,0	2,5	2,0	2,0	0,0	
	Opponent MP					4	7	10	5	6	6	6	
	ESB Contribute					8,0	14,0	10,0	12,5	12,0	12,0	0,0	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMGSB
9	Iguanas	6	14½	6	14½	2w1	12b4	3b1	14w3	7w2	8b2	4w1½	83,0
	Team GP					1,0	4,0	1,0	3,0	2,0	2,0	1,5	
	Opponent MP					10	2	10	4	7	7	10	
	ESB Contribute					10,0	8,0	10,0	12,0	14,0	14,0	15,0	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMGSB
13	Moose	6	11½	6	11½	6w2	14b2½	4w1	11w2½	2b1	7b2	5w½	73,0
	Team GP					2,0	2,5	1,0	2,5	1,0	2,0	0,5	
	Opponent MP					7	4	10	4	10	7	10	
	ESB Contribute					14,0	10,0	10,0	10,0	10,0	14,0	5,0	

Since in this tie-break the contribute of an opponent is given by its total MP score multiplied by the GP score obtained against it, two teams that have won (or lost) against the same opponent may have a different contribution based on how well they won (or how poorly they lost). For example, teams #9 and #13 both played and won against team #14, but team #9 won with a 3-1 score, so the contribution it gets from this match is $3 \times 4 = 12$ points, while team #13 won with a $2\frac{1}{2} - 1\frac{1}{2}$ score, resulting in a contribution of $2.5 \times 4 = 10$ points. It may happen that this opponent giving different contributes to the two teams is just the one giving the least significant contribute. In this case, when applying the Cut-1 modifier, we are cutting contributes that are not all equal (in contrast to what happened in the previous examples, for instance, with the EMMSB variant). The application of the Cut-1 modifier may therefore have different effects for the two (or possibly more) teams that faced a same "least significant" opponent.

Incidentally, it is also worth noting that here the score used for the team in calculating the tie-break is the secondary score and not the primary one. Although this may seem "strange" at first glance, there is in fact no drawback to doing so.

Exercise 41

Primary score is MP. Calculate EGGSB for all teams at 7 points. Resolve residual ties using the EGGSB-Cut 1 system.

Let's extract from the general crosstable the data relevant to the tied teams (see previous examples). For this calculation, we need GP scores of the involved teams and of their

opponents. Once again, the calculation method is always the same.

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EGGSB
6	Falcons	7	12½	7	12½	13b2	5w1	8b3	3w1½	11b2½	-F	12w2½	157,50
	<i>Team GP</i>					2,0	1,0	3,0	1,5	2,5	0,0	2,5	
	<i>Opponent GP</i>					11,5	18,0	15,0	16,0	11,5	12,5	7,5	
	<i>ESB Contribute</i>					23,00	18,00	45,00	24,00	28,75	0,00	18,75	

#	TEAM	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EGGSB
8	Hippopotami	7	15	7	15	1b1½	7w2	6w1	12b3½	10b2	9w2	11w3	181,50
	<i>Team GP</i>					1,5	2,0	1,0	3,5	2,0	2,0	3,0	
	<i>Opponent GP</i>					17,5	13,5	12,5	7,5	13,0	14,5	11,5	
	<i>ESB Contribute</i>					26,25	27,00	12,50	26,25	26,00	29,00	34,50	

The possible values for this tie-break are highly varied, making differences very probable. Among the four variants of the ESB system, this is the one that provides the best resolution of the ranking (although this doesn't necessarily imply the highest reliability). It's worth noting that the numbers are large (because a result expressed in GP is numerically larger) and, to represent them correctly, two decimal places are necessary.

10.1 The Olympiad tie-break

The tie-break regulations ([16.6]) explicitly allow the competition rules to provide for a different treatment of unplayed matches. This happens, for example, in the Olympics. Let's take, for instance, the main tie-break indicated for the 2026 edition. Examining the regulations (see *FIDE Handbook, D.02.01, Appendix 2*), we see that this tie-break is rather similar to the Sonneborn-Berger, EMGSB Cut-1 variant.

However, in addition to the slightly different definition of unplayed rounds, the cutting procedure here is different: the preferred choice is the round in which the team received the PAB. Only if the team did not receive a PAB, the contribution from the opponent with the lowest match points score is excluded.

To conclude the chapter, the reader is encouraged to practice calculating other tie-breaks. To help the reader to verify their results, the following table reports the tie-break values calculated with each of the four variants.

#	Squadra	MP	GP	Adj MP	Adj GP	R1	R2	R3	R4	R5	R6	R7	EMM SB	EGM SB	EMG SB	EGG SB
1	Antelopes	10	17½	10	17½	8w2½	4b1½	7w3	2b2½	5w1½	+F	3b2½	88	158,0	158,5	283,00
2	Bonobos	10	17	10	17	9b3	3w4	5b0	1w1½	13w3	4b2½	10b3	74	144,0	131,0	249,75
3	Cougars	10	16	10	16	10w2½	2b0	9w3	6b2½	4w3	5b3½	1w1½	76	150,0	128,0	247,50
4	Deer	10	17	10	17	11b3	1w2½	13b3	5w3½	3b1	2w1½	9b2½	72	146,0	130,0	253,50
5	Elephants	10	18	10	18	12w4	6b3	2w4	4b½	1b2½	3w½	13b3½	70	132,0	125,0	236,00
6	Falcons	7	12½	7	12½	13b2	5w1	8b3	3w1½	11b2½	-F	12w2½	32	79,5	73,0	157,50
7	Giraffes	6	11½	7	13½	14w2	8b2	1b1	10w2½	9b2	13w2	ZPB	33	78,5	68,5	155,00
8	Hippopotami	7	15	7	15	1b1½	7w2	6w1	12b3½	10b2	9w2	11w3	30	79,0	77,0	181,50
9	Iguanas	6	14½	6	14½	2w1	12b4	3b1	14w3	7w2	8b2	4w1½	26	66,5	83,0	180,00
10	Jackals	5	13	5	13	3b1½	11w1½	12b2½	7b1½	8w2	14b3	2w1	19	53,0	72,5	161,75
11	Koalas	4	11½	4	11½	4w1	10b2½	14w2	13b1½	6w1½	12w2	8b1	16	45,0	61,0	138,50
12	Lynxes	2	7½	2	7½	5b0	9w0	10w1½	8w½	PAB	11b2	6b1½	6	19,0	33,5	83,75
13	Moose	6	11½	6	11½	6w2	14b2½	4w1	11w2½	2b1	7b2	5w½	30	72,0	73,0	152,50
14	Narwhals	4	11½	4	11½	7b2	13w1½	11b2	9b1	HPB	10w1	PAB	19	48,0	58,0	140,75

11 EXTENDED DIRECT ENCOUNTER FOR TEAMS (EDE)

The direct encounter tie-break for team tournaments [13.3] is rather complex, but it takes into account that some of the involved teams may not have faced each other. Additionally, it considers the possibility of using the secondary score when the primary score fails to resolve all ties. This tie-break is currently the only one that can appear multiple times in the list of tie-breaks [4.1]. For example, we could have the list {EDE, EGMSB, EDE}, where the remaining ties after the first application of direct encounter are resolved using Sonneborn-Berger, and any further remaining ties are again resolved using EDE.

In Swiss-system tournaments, unplayed matches are all ignored. On the contrary, in tournaments with predetermined pairings (round-robin, Scheveningen, Schiller, etc.), forfeits are treated just as played matches [15.2] – although tournament rules may still establish different behaviours.

The direct encounter tie-break is applied in multiple, possibly repeated, phases, as illustrated below.

1. First, a separate crosstable is created with only the matches between the tied teams, excluding those involving teams with different scores.
2. From this, a ranking is prepared for the relevant teams only ("separate ranking"), using the primary score.

In Swiss-system tournaments, some teams may not have faced each other, resulting in gaps in the separate crosstable. In certain cases, a team might inevitably rank first, regardless of any possible outcomes of the missing matches. In this case, the team is indeed ranked first, and we proceed to check if the same holds for a possible second place (and so on).

3. If all teams are still tied, the entire process is repeated using the secondary score instead of the primary one.
 - a. If exactly two teams remain tied, the tournament rules may specify the application of one or more tie-breaks specific to direct elimination tournaments [12] (which are generally suitable for team tournaments as well).
4. If more than two teams remain tied, a new separate crosstable is compiled with only the teams still tied, and the process starts anew.

The outlined procedure shows how the direct encounter tie-break can be applied multiple times to progressively smaller groups of tied teams – until no further ties can be resolved, and the next tie-break is invoked.

Exercise 42

Primary score is MP. Rank all teams at 4 points using the EDE system.

First things first, we need to extract the data of the involved teams from the crosstable.

#	TEAM	MP	GP	R1	R2	R3	R4	R5	R6	R7
11	Koalas	4	11½	4w1	10b2½	14w2	13b1½	6w1½	12w2	8b1
14	Narwhals	4	11½	7b2	13w1½	11b2	9b1	HPB	10w1	PAB

Since there are only two teams involved, this once only there is no need to physically compile a separate crosstable. The results show that the two teams played with each other and drew, remaining tied. Therefore, we must compare the secondary score, but it is also equal. Hence, we should move on to the next tie-break.

Exercise 43

Primary score is MP. Rank all teams at 7 points using the EDE system.

Let's start by extracting the data for the involved teams from the general crosstable.

#	TEAM	MP	GP	R1	R2	R3	R4	R5	R6	R7
6	Falcons	7	12½	13b2	5w1	8b3	3w1½	11b2½	-F	12w2½
8	Hippopotami	7	15	1b1½	7w2	6w1	12b3½	10b2	9w2	11w3

Once again, there are only two teams involved. Now, however, the teams played each other and team #6 won – hence, it precedes team #8 in the standings. Incidentally, we observe that a tie-break system based on secondary score, like MPvGP or some variants of Sonneborn-Berger (EMGSB, EGGSB) would yield the opposite result.

Exercise 44

Primary score is MP. Rank all teams at 6 points using the EDE system.

Again, let's extract from the general crosstable the data for the involved teams.

#	TEAM	MP	GP	R1	R2	R3	R4	R5	R6	R7
7	Giraffes	6	11½	14w2	8b2	1b1	10w2½	9b2	13w2	ZPB
9	Iguanas	6	14½	2w1	12b4	3b1	14w3	7w2	8b2	4w1½
13	Moose	6	11½	6w2	14b2½	4w1	11w2½	2b1	7b2	5w½

In this case, it is advisable to explicitly compile the separate crosstable (on the right – game points (GP) obtained in the match are reported). The ranking, formulated based on the primary score (MP), sees #7 with two (MP) points, while #9 and #13 are tied at one point, but they have not played against each other. Therefore, we need to consider the possible outcomes of this match. For example, if #9 had won, it would move to three points, surpassing team #7. So, nothing can be said about this ranking. If everyone remains tied, we must move on to the alternative score (GP) – but even in this case, nothing changes. In conclusion, once again EDE cannot determine the ranking order.

	7	9	13
7	*	B2	W2
9	W2	*	-
13	B2	-	*

Exercise 45

Primary score is MP. Rank all teams at 10 points using the EDE system.

Again, let's extract from the general crosstable the data for the five involved teams.

#	TEAM	MP	GP	R1	R2	R3	R4	R5	R6	R7
1	Antelopes	10	17½	8w2½	4b1½	7w3	2b2½	5w1½	+F	3b2½
2	Bonobos	10	17	9b3	3w4	5b0	1w1½	13w3	4b2½	10b3
3	Cougars	10	16	10w2½	2b0	9w3	6b2½	4w3	5b3½	1w1½
4	Deer	10	17	11b3	1w2½	13b3	5w3½	3b1	2w1½	9b2½
5	Elephants	10	18	12w4	6b3	2w4	4b½	1b2½	3w½	13b3½

All teams played each other (this does not happen often...) so we can compile a complete separate crosstable.

This case is a bit more complicated, so we'll follow the procedure step by step. First, with the data from the general crosstable, we compose the separate crosstable (on the right) and calculate the MP scores of the teams.

	1	2	3	4	5	MP	GP
1	*	B2½	B2½	B1½	W1½	4	8,0
2	W1½	*	W4	B2½	B0	4	8,0
3	W1½	B0	*	W3	B3½	4	8,0
4	W2½	W1½	B1	*	W3½	4	8,5
5	B2½	W4	W½	B½	*	4	7,5

Since these are all the same, the teams are still all tied and we must therefore try again, this time using the GP scores [13.3.1]. Doing so, we succeed in assigning the first place to team #4 and the fifth to #5, while the other three teams remain still tied.

We must therefore apply again the direct encounter, to the three remaining teams only, *still using the GP because we are still within the same application of the tie-break*, as indicated by [6], endnote. This time the scores are different, so they can determine the three positions in the ranking.

	1	2	3	GP
1	*	B2½	B2½	5,0
2	W1½	*	W4	5,5
3	W1½	B0	*	1,5

We thus arrive at the final ranking (on the right). It's worth taking a moment to consider the resulting ranking in light of the total scores obtained by the teams. In fact, a typical aspect of the direct encounter tie-break becomes evident here – team #2 is positioned ahead of team #1, *despite having a lower GP score*. Furthermore, the team with the highest GP score even ranks last, after the team with the lowest GP score!

	Crosstable		EDE		
	MP	GP	MP 1	GP 2	GP 3
4	10	17	4	8,5	---
2	10	17	4	8,0	5,5
1	10	17½	4	8,0	5,0
3	10	16	4	8,0	1,5
5	10	18	4	7,5	---

The above examples suggest that the direct encounter can rarely differentiate between tied positions. However, there is a widespread (although debatable) sentiment that it is a "fair" tie-break because it favours the team who defeated the others. As with all tie-breaks, the matter is philosophical, and it is always up to the tournament organizer to decide which tie-break strategy they consider better (and for the players to approve or disapprove, by participating or not in the tournament). We conclude, as usual, with a comparison between podium positions obtained by using various tie-break systems.

TEAM		PUNTEGGI				SONNEBORN-BERGER				BUCHHOLZ				EDE
#	Team	MP	GP	Adj MP	Adj GP	EMM	EGM	EMG	EGG	BH MP	C1	BH GP	C1	
1	Antelopes	10	17½	10	17½	1	1	1	1	1	1	1	1	3
2	Bonobos	10	17	10	17	3	4	2	3	3	4	3	3	2
3	Cougars	10	16	10	16	2	2	4	4	2	2	2	2	4
4	Deer	10	17	10	17	4	3	3	2	4	4	4	4	1
5	Elephants	10	18	10	18	5	5	5	5	5	2	5	5	5

12 SYSTEMS BASED ON BOARDS COUNT

The tie-breaks in this group are meant to be used in team knockout tournaments, when the tied teams have equal match and game scores. Individual forfeits are considered equivalent to actually played matches, and PABs (Pairing Allocated Byes) provide the same points as a regular win. The rules of an event may specify the use of one or more of these tie-breaks in any team tournament. Additionally, they may indicate their use in association with extended direct encounter tie-breaks when only two teams remain tied.

To apply these tie-breaks, knowledge of the player line-ups in the teams is necessary. Such information that does not appear in the crosstable and must be obtained from other sources (e.g., detailed pairings).

12.1 Board count (BC)

The board count is an example of score weighted according to the board position. The contribution of each board is given by the product of the result achieved on that board (regardless of the player) multiplied by the number of the board itself. The value of the tie-break is the sum of all these contributions (which usually are four).

To illustrate the behaviour of this tie-break, the following table shows the value of BC for each possible result of a team (of four players) that achieved a draw (1 MP, 2 GP).

B1	0	0	0	0	0,5	0	0,5	0,5	0	0,5	1	0,5	0,5	1	0,5	1	1	1	1
B2	0	0,5	0,5	1	0	1	0	0,5	1	0,5	0	0,5	1	0	1	0	0,5	0,5	1
B3	1	0,5	1	0	0,5	0,5	1	0	1	0,5	0	1	0	0,5	0,5	1	0	0,5	0
B4	1	1	0,5	1	1	0,5	0,5	1	0	0,5	1	0	0,5	0,5	0	0	0,5	0	0
BC	7,0	6,5	6,0	6,0	6,0	5,5	5,5	5,5	5,0	5,0	5,0	4,5	4,5	4,5	4,0	4,0	4,0	3,5	3,0

It is readily apparent that the result is lower the higher the boards on which the result was achieved. Since the underlying idea of this tie-break is to give more importance to the first board, decreasing gradually towards the last, it follows that the lower the total, the better the placement.

The same BC value can be obtained with different scores (for example, a BC value of 5 can correspond to a match lost, drawn, or won) – hence:

this tie-break can only be used if all teams have the same MP.

Exercise 46

Primary score is MP. The first criterion in the tie-break list is EDE system with board count [13.3.2]. Establish the ranking order of teams #11 and #14.

Let's extract from the general crosstable the data for the involved teams.

#	TEAM	MP	GP	R1	R2	R3	R4	R5	R6	R7
11	Koalas	4	11½	4w1	10b2½	14w2	13b1½	6w1½	12w2	8b1
14	Narwhals	4	11½	7b2	13w1½	11b2	9b1	HPB	10w1	PAB

In the direct encounter match, which occurred in the third round, the teams drew, and the application of [13.3.1], first with MP and then with GP, leaves the teams still tied. We are thus in the scenario described by [13.3.2], and the tournament rules specify the application of BC. We need the line-ups of the two teams in the match, which we can derive from the pairing:

Table	id	team	result	team	id
5	11	Koalas	2-2	Narwhals	14

Board	id	player	result	player	id
1	33	Kris Kelpa	1-0	Nikola Neric	34
2	43	Kelly Kort	½-½	Noah Negus	55
3	52	Kirk Koman	½-½	Nicola Neba	37
4	73	Kurt Kontos	0-1	Nuccio Negri	62

Now we can calculate the tie-break values.

$$BC (\#11) = 1 \times \mathbf{1} + 0,5 \times \mathbf{2} + 0,5 \times \mathbf{3} + 0 \times \mathbf{4} = 1 + 1 + 1,5 + 0 = 3,5$$

$$BC (\#14) = 0 \times \mathbf{1} + 0,5 \times \mathbf{2} + 0,5 \times \mathbf{3} + 1 \times \mathbf{4} = 0 + 1 + 1,5 + 4 = 6,5$$

Finally, we compare the results to establish that the value for team #14 is higher, determining the precedence of team #11.

Incidentally, in this particular case, using the principle of giving more importance to the higher boards, one could have predicted the tie-break outcome simply by observing the results. However, a careful examination of the table shown at the beginning of the paragraph reveals that things are not always so straightforward.

12.2 Top board results (TBR)

This tie-break considers the results obtained on the higher boards (in only actually played games, regardless of the player), starting from the first board alone and then gradually extending to the lower ones until the tie is resolved. This tie-break can also be considered a kind of weighted average of scores per board, where the weights are progressively adjusted in case of persistent ties.

Exercise 47

Primary score is MP. The first criterion in the tie-break list is EDE system with top board results [13.3.2]. Establish the ranking order of teams #11 and #14.

As seen in the previous example, the teams drew, and the application of [13.3.1] leaves them tied, so we move on to [13.3.2], where this time the TBR is applied. The formations of the two teams are needed again, which we report below.

Table	id	team	result	team	id
5	11	Koalas	2-2	Narwhals	14
Board	id	player	result	player	id
1	33	Kris Kelpa	1-0	Nikola Neric	34
2	43	Kelly Kort	1/2-1/2	Noah Negus	55
3	52	Kirk Koman	1/2-1/2	Nicola Neba	37
4	73	Kurt Kontos	0-1	Nuccio Negri	62

There is nothing to calculate since the application is immediate. On the first board, team #11 won, thus prevailing over the opponent. It's worth noting that this tie-break, although different from the previous one, has similar practical effects.

12.3 Bottom board elimination (BBE)

This tie-break is, in a sense, complementary to the previous one. If before we considered the top boards, now we progressively eliminate the last ones. This shifts the focus from the (usually) stronger players of the team to the (usually) weaker ones. Both tie-breaks favour the results of the higher boards, thus giving better results for teams whose players are lined up in order of playing strength.

Exercise 48

Primary score is MP. The first criterion in the tie-break list is EDE system with bottom board elimination [13.3.2]. Establish the ranking order of teams #11 and #14.

As mentioned above, applying [13.3.1] leaves those teams still tied and we must resort to [13.3.2], where BBE is used. Once again, we need lineups.

Table	id	team	result	team	id
5	11	Koalas	2-2	Narwhals	14
Board	id	player	result	player	id
1	33	Kris Kelpa	1-0	Nikola Neric	34
2	43	Kelly Kort	1/2-1/2	Noah Negus	55
3	52	Kirk Koman	1/2-1/2	Nicola Neba	37
4	73	Kurt Kontos	0-1	Nuccio Negri	62

The application is straightforward.

$$\text{BBE} (\#11) = 1 + 0,5 + 0,5 = 2$$

$$\text{BBE} (\#14) = 0 + 0,5 + 0,5 = 1$$

Once again, team #11 prevail over the opponent.

13 SCORES AND SCHEDULE STRENGTH COMBINATION (SSSC)

This is a somewhat intricate tie-break that considers both the team's secondary score and the strength of the encountered opposition at the same time, providing an estimate of the team's actual playing strength. However, calculating it is not too difficult, requiring in practice only a few more operations than the Buchholz, on which it partly relies.

The tie-break value is the sum of two elements. The first is simply the team's secondary score (GP if the primary score is MP, or vice versa), representing the overall results.

The second term, which assesses the strength of the opposition, uses the Buchholz system calculated based on the primary score. However, the Buchholz value can be numerically much greater than the secondary score and therefore, in order to balance the two terms, it needs to be normalized. This is achieved by dividing it by a *normalisation factor*, which essentially accounts for the imbalance between the two. The value of this factor is the same for all teams and is calculated by dividing the *maximum possible primary score* (which depends on the number of rounds) by the *maximum secondary score in a match* (which depends on the number of players in the team) and rounding down. Tournament regulations may specify a different normalisation value.

It's worth noting that, since the Buchholz manages unplayed matches (replacing them with "dummy" matches), the SSSC system implicitly does the same.

Example 1: Let's consider a tournament with eleven rounds and teams of four members, with MP as primary score (following the Olympic format). The maximum possible team score is given by the win score (2 MP) multiplied by the number of rounds (11), resulting in 22 MP. The maximum secondary score achievable in a round is equal to the number of players per team, which is 4. The quotient between these values is $22/4 = 5.5$. Rounding down ([13.4.2.b]) this quotient, we obtain the normalization factor $F_N=5$.

Example 2: Let's consider a tournament with nine rounds and teams of four members, with GP as primary score. The maximum possible team score is given by the maximum score per match (4 GP) multiplied by 9 rounds, resulting in 36 GP. The maximum secondary score achievable in a round is equal to the victory score (2 MP). Therefore, the normalization factor is $F_N = 36/2 = 18$ (there's no need to round in this case).

What is the meaning of the normalisation factor?

To better focus on the significance of this factor, we want to estimate the ratio between the two terms that make up the SSSC tie-break. For simplicity, let's consider the case where the primary score is MP, and conventional scoring systems are used (2-1-0 for matches, 1-1/2-0 for games).

The maximum secondary score in a round, GP_{MAXR} , is simply equal to the number N_G of players fielded per team.

If N_R is the number of rounds, the maximum secondary score in this tournament is therefore:

$$GP_{MAX} = N_R \times N_G$$

The maximum primary score, MP_{MAX} , is equal to the score S_W assigned for a match win multiplied by the number N_R of rounds:

$$MP_{MAX} = N_R \times S_W$$

The maximum possible Buchholz, which would be obtained by facing all opponents with a perfect score, is therefore given by the maximum primary score multiplied by the number of matches:

$$BH_{MAX} = N_R \times MP_{MAX} = N_R \times N_R \times S_W.$$

Using these boundary values, we can estimate the ratio R between the two components of the tie-break as

$$R = BH_{MAX} / GP_{MAX} = (N_R \times N_R \times S_W) / (N_R \times N_G), \text{ or}$$

$$R = (N_R \times S_W) / N_G = MP_{MAX} / GP_{MAXR}$$

Apart from rounding (imposed for simplicity), this last expression is indeed equal to the normalization factor. We can therefore attribute it a meaning as the F_N factor, such that $GP_{MAX} = BH_{MAX} / F_N$. Hence, *this factor makes the value of the secondary score GP_{TOT} comparable to that of the Buchholz.*

Of course, the reasoning could be repeated with another primary score or for other scoring systems, leading to similar conclusions.

Exercise 49

Primary score is MP. Calculate the SSSC tie-break value for all teams.

First, we calculate the normalization factor. The tournament has seven rounds, and the match win score is two MP. Therefore, the maximum primary score achievable in the entire tournament is $S_{MP} = 2 \text{ MP} \times 7 \text{ rounds} = 14 \text{ MP}$. The maximum secondary score achievable in one round is, of course, one point for each player fielded, so in our case, it is 4 GP. Dividing the former by the latter and rounding, we obtain the value of the normalization factor, which here is 3.

MAX PRIMARY SCORE (TOTAL)	MAX SECONDARY SCORE (ROUND)	QUOTIENT	NORMALISATION FACTOR
14	4	3,5	3

Using the values already calculated for the Buchholz (see exercise 35) and this normalization factor, we can now easily calculate the tie-break for each team. As an example, let's calculate it for team #1. The opposition is given by the Buchholz (calculated on the primary score) divided by the normalization factor: $OPP_1 = BH_1 / F_N = 64 / 3 \approx 21.3$ (for our convenience, the result is rounded to one decimal place). By adding this term to the final GP score of the team, we get $SSSC_1 = GP_1 + OPP_1 = 17.5 + 21.3 = 38.8$.

Proceeding similarly for the other teams, we can compile the following table.

#	TEAM	MP (PRI)	GP (SEC)	BH (MP)	OPPOSITION	SSSC
1	Antelopes	10	17,5	64	21,3	38,8
2	Bonobos	10	17,0	57	19,0	36,0
3	Cougars	10	16,0	58	19,3	35,3
4	Deer	10	17,0	56	18,7	35,7
5	Elephants	10	18,0	55	18,3	36,3
6	Falcons	7	12,5	46	15,3	27,8
7	Giraffes	6	11,5	44	14,7	26,2
8	Hippopotami	7	15,0	41	13,7	28,7
9	Iguanas	6	14,5	50	16,7	31,2
10	Jackals	5	13,0	44	14,7	27,7
11	Koalas	4	11,5	41	13,7	25,2
12	Lynxes	2	7,5	41	13,7	21,2
13	Moose	6	11,5	52	17,3	28,8
14	Narwhals	4	11,5	36	12,0	23,5

Let's conclude this chapter by comparing the podium positions given by the various tie-breaks.

TEAM		SCORES		SONNEBORN-BERGER				BUCHHOLZ				EDE	SSSC
#	Team	MP	GP	EMM	EGM	EMG	EGG	BH MP	C1	BH GP	C1		
1	Antelopes	10	17½	1	1	1	1	1	1	1	1	3	1
2	Bonobos	10	17	3	4	2	3	3	4	3	3	2	3
3	Cougars	10	16	2	2	4	4	2	2	2	2	4	5
4	Deer	10	17	4	3	3	2	4	4	4	4	1	4
5	Elephants	10	18	5	5	5	5	5	2	5	5	5	2

It is worth noting that SSSC rewards with the second place the high game score of team #5, which almost all other tie-breaks would assign to the fifth place (whereas the MPvGP system would even place it in the first position). Conversely, it gives the fifth place to the relatively low score of team #3, which other tie-breaks position at the second place.